CBSE Test Paper 02

CH-09 Sequences and Series

1. The product of first n odd terms of a G.P. whose middle term is m is
   a. none of these
   b. m^n
   c. n^m
   d. mn
2. Sum of an infinitely many terms of a G.P. is 3 times the sum of even terms. The common ratio of the G.P. is
   a. 2
   b. $\frac{3}{2}$
   c. none of these
   d. $\frac{1}{2}$
3. The sum of terms equidistant from the beginning and end in A.P. is equal to
   a. last term
   b. 0
   c. first term
   d. sum of the first and the last terms
4. If $a \in R$, then the roots of the equation tan x = a are in G.P for what values of a
   a. $\frac{1}{\sqrt{3}}, 1, \sqrt{3}$
   b. 1,0,-1
   c. H.P.
   d. none of these
5. If a, b, c are in A. P. as well as in G.P.; then
   a. $a = b \neq c$
   b. $a \neq b = c$
   c. a = b = c
   d. $a \neq b \neq c$
6. Fill in the blanks:

   If $\sum n = 210$, then $\sum n^2$ = _______.

7. Fill in the blanks:

   A.M between x - 3 and x + 5 is ________.

8. Find the three terms of an AP whose sum is 9 and common difference is 1.

9. Which term of the sequence $\sqrt{3}, 3, 3\sqrt{3}$, ... is 729?

10. Which term of the sequence $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$,...... is $\frac{1}{19683}$?

11. Let the sequence a_n is defined as follows $a_1 = 2$, $a_n = a_{n-1} + 3$ for $n \geq 2$. Find the first five terms and write corresponding series.

12. For what values of x, the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.?

13. If a, b, c are in A.P.; b, c, d are in G.P. and $\frac{1}{c}$, $\frac{1}{d}$, $\frac{1}{e}$ are in A.P., prove that a, c, e are in G.P.

14. In an A.P., if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(pq+1)$, where $p \neq q$.

15. If in an A.P. the sum of m terms is equal to n and the sum of n terms is equal to m, then prove that the sum of(m + n) terms is - (m + n). Also, find the sum of the first (m - n) terms (m > n).

Solution

1. (b) m^n

   Explanation:

   Let the terms in a GP be a,ar,$a, ar, ar^2, ar^3, \ldots\ldots\ldots$
   Product of odd number of terms (let number of terms be 'n' , n is odd)
   $= a \cdot ar \cdot ar^2 \cdot ar^3 \ldots\ldots\ldots\ldots ar^{n-1}$
   $= a^n \cdot \left(1 \cdot r \cdot r^2 \cdot r^3 \ldots\ldots\ldots\ldots r^{n-1}\right)$
   $= a^n, r^{1+2+3+\ldots\ldots\ldots\cdots+n-1}$
   $= a^n \cdot r^{\frac{(n-1)n}{2}}$ [The sum of first n natural numbers = $\frac{n(n+1)}{2}$]
   We have the middle term $= m = ar^{\frac{(n-1)}{2}}$ ......(i)
   $\therefore$ Product of odd number of terms = $a^n \cdot r^{\frac{n(n-1)}{2}} = \left(ar^{\frac{(n-1)}{2}}\right)^n$ = m^n [using (i)]

2. (d) $\frac{1}{2}$

   Explanation:
   Consider the infinite G . P $a, ar, ar^2, ar^3, \ldots\ldots\ldots\ldots\ldots, \ldots$ with first term a and common ratio r
   Then the even terms $ar, ar^3, ar^5, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ is again an infinite G.P with first term ar and common ratio r^2
   W e have $S_\infty = \frac{a}{1-r}$
   Given $S_\infty = 3$. Sum of even terms
   $\Rightarrow a + ar + ar^2 + ar^3 +$.........$= 3. \left[ar + ar^3 + ar^5 +\right.$............]
   $\Rightarrow \frac{a}{1-r} = 3 \cdot \frac{ar}{1-r^2}$
   $\Rightarrow \frac{1}{1-r} = 3 \cdot \frac{r}{(1-r)(1+r)}$
   $\Rightarrow 1(1+r) = 3.r$
   $\Rightarrow 2r = 1 \Rightarrow r = \frac{1}{2}$

3. (d) sum of the first and the last terms

Explanation:

Let the first term of the A.P be a , last term be l and the common difference be d

Now the A.P will be of the form $a, a+d, a+2d, \ldots\ldots\ldots, l-2d$,l-d,l

Sum of two term equidistant from the beggining and end (say r+1 t term) = a+r d+b-r d=a+b = sum of the first term and last term

4. (a) $\frac{1}{\sqrt{3}}, 1, \sqrt{3}$

   Explanation:

   We have $\tan 30^{\circ}=\frac{1}{\sqrt{3}}, \tan 45^{\circ}=1$ and $\tan 60^{\circ}=\sqrt{3}$

   Also we have $\frac{1}{\sqrt{3}}, 1, \sqrt{3}$ are in G.P

5. (c) a = b = c

   Explanation:

   a,b,c are in A.P, $2b=a+c$.......................(i)

   a,b,c are in G.P, $b^2=ac$............................(ii)

   from (i) and (ii), we get

   $\left(\frac{a+c}{2}\right)^2=ac$

   $\Rightarrow (a+c)^2-4ac=0$

   $\Rightarrow (a-c)^2=0 \Rightarrow a=c$

   using a=c in (ii)

   $2b=c+c$

   $\Rightarrow b=c$

   so, a=b=c

6. 2870

7. x + 1

8. Let the three terms of AP are a - d, a and a + d.

   ∴ a - d + a + a + d = 9

   ⇒ 3a = 9 ⇒ a = 3

   Also, d = 1 [given]

   ∴ Required terms are 3 - 1, 3, 3 + 1

i.e. 2, 3, 4

9. Here a = $\sqrt{3}$, r = $\frac{3}{\sqrt{3}} = \sqrt{3}$ and a_n = 729

$\therefore a_n = ar^{n-1}$

$\Rightarrow 729 = \sqrt{3} \times (\sqrt{3})^{n-1}$

$\Rightarrow (\sqrt{3})^{12} = (\sqrt{3})^{n}$

$\Rightarrow$n = 12

Therefore, 12^{th} term of the given G.P. is 729.

10. Here a = $\frac{1}{3}$, r = $\frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$ and a_n = $\frac{1}{19683}$

$\therefore a_n = ar^{n-1}$

$\Rightarrow \frac{1}{19683} = \frac{1}{3} \times \left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow \left(\frac{1}{3}\right)^{9} = \left(\frac{1}{3}\right)^{n}$

$\Rightarrow$n = 9

Therefore, 9^{th} term of the given G.P. is $\frac{1}{19683}$

11. We have, a_1 = 2, and $a_n = a_{n-1} + 3$

On putting n = 2, we get

$a_2 = a_1 + 3 = 2 + 3 = 5$

On putting n = 3, we get

$a_3 = a_2 + 3 = 5 + 3 = 8$

On putting n = 4, we get

$a_4 = a_3 + 3 = 8 + 3 = 11$

On putting n = 5, we get

$a_5 = a_4 + 3 = 11 + 3 = 14$

Thus, first five terms of given sequence are 2, 5, 8, 11 and 14.

Also, corresponding series is 2, 5, 8, 11 , 14 , 17.......

12. Given, $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

$\therefore \frac{x}{\frac{-2}{7}} = \frac{\frac{-7}{2}}{x}$

$\Rightarrow x^2 = \frac{-2}{7} \times \frac{-7}{2}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

Therefore, for x = ± 1 th given numbers are in G.P.

13. Since, a, b, c are in A.P.

$\therefore$ b - a = c - b

$\Rightarrow$ 2b = a + c

$\Rightarrow b = \frac{a+c}{2}$

Since, b, c, d are in G.P.

$\therefore \frac{c}{b} = \frac{d}{c}$

$\Rightarrow$ c^2 = bd..........(i)

Also $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P.

$\therefore \frac{1}{d} - \frac{1}{c} = \frac{1}{e} - \frac{1}{d}$

$\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$

$\Rightarrow \frac{2}{d} = \frac{c+e}{ce}$

$\Rightarrow d = \frac{2ce}{c+e}$

Putting values of b and d in eq. (i), $c^2 = \left(\frac{c+a}{2}\right)\left(\frac{2ce}{c+e}\right)$

$\Rightarrow c^2 = \frac{\text{ce}(c+a)}{c+e}$

$\Rightarrow$ c^2(c + e) = ec(c + a)

$\Rightarrow$ c^2 + ce = ce + ae

$\Rightarrow$ c^2 = ae which shows that a, c, e are in G.P.

14. Let a be the first term and d be the common difference of given A.P.

And $a_p = \frac{1}{q}$ and $a_q = \frac{1}{p}$

$\therefore a + (p-1)d = \frac{1}{q}$ and $a + (q-1)d = \frac{1}{p}$

$\Rightarrow a + pd - d = \frac{1}{q}$ .....(i) and $a + qd - d = \frac{1}{p}$ .....(ii)

Subtracting eq. (ii) from eq. (i), we get

a + pd - d - (a + qd - d) $= \frac{1}{q} - \frac{1}{p}$

$\Rightarrow$ pd - d - a - qd + d $= \frac{p-q}{pq}$

$\Rightarrow (p-q)d = \frac{p-q}{pq}$

$\Rightarrow d = \frac{p-q}{pq} \times \frac{1}{p-q} = \frac{1}{pq}$

Putting value of d in eq. (i), we get

$a + p\frac{1}{pq} - d = \frac{1}{q}$

$\Rightarrow a + \frac{1}{q} - d = \frac{1}{q}$

$\Rightarrow a = \frac{1}{q} + d - \frac{1}{q} = d = \frac{1}{pq}$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{pq} = \frac{pq}{2}\left[2 \times \frac{1}{pq} + (pq-1) \times \frac{1}{pq}\right]$

$\Rightarrow S_{pq} = \frac{pq}{2}\left[\frac{2}{pq} + \frac{pq-1}{pq}\right]$

$\Rightarrow S_{pq} = \frac{pq}{2}\left[\frac{2+pq-1}{pq}\right]$

$\Rightarrow S_{pq} = \frac{pq}{2}\left[\frac{1+pq}{pq}\right]_= \frac{pq+1}{2}$

$\Rightarrow S_{pq} = \frac{1}{2}(pq+1)$

15. Let a be the first term and d be the common difference of the given A.P. Then,
S_m = n $\Rightarrow$ $\frac{m}{2}$ {2a + (m − 1)d} = n $\Rightarrow$ 2am + m (m− 1) d = 2n .....(i)
and $\Rightarrow$ S_n = m $\Rightarrow$ $\frac{m}{2}$ {2a + (m − 1) d} = m $\Rightarrow$ 2an + n (n -1) d = 2m .....(ii)

Subtracting (ii) from (i), we get

2a (m - n) + {m(m -1) - n(n - 1)} d = 2n - 2m

$\Rightarrow$ 2a (m - n) + {(m^2 - n^2) - (m - n)} d = - 2 (m - n)

$\Rightarrow$ 2a + (m + n -1) d = - 2 [On dividing both sides by (m - n)] ...(iii)

Now, $S_{m+n} = \frac{m+n}{2}$ {2a + (m + n -1) d}

$\Rightarrow S_{m+n} = \frac{(m+n)}{2}$ (- 2) [Using (iii)]

$\therefore$ Sm + n = (m + n)

From (iii), we obtain

2a = - 2 - (m + n - 1) d ....(iv)

Substituting this value of 2a in (i), we obtain

- 2m - m (m + n - 1) d + m (m - 1) d = 2n

$\Rightarrow$ d = - 2 $\left(\frac{m+n}{mn}\right)$ ....(v)

Putting d = - 2 $\left(\frac{m+n}{mn}\right)$ in (iv), we obtain

2a = -2 + $\frac{2}{mn}$ (m + n -1) (m + n) ....(vi)

Now,

$S_{m-n} = \frac{m-n}{2}$ {2a + (m - n -1) d}

$\Rightarrow S_{m-n} = \frac{m-n}{2}$ {-2 + $\frac{2}{mn}$ (m + n - 1) (m + n) - $\frac{2}{mn}$ (m - n - 1) (m + n)} [Using (v) and (vi)]

$\Rightarrow S_{m-n}$ = {- 2 + $\frac{4n}{mn}$ (m + n)} = $\frac{1}{m}$ (m - n) (m + 2n)