CBSE Test Paper 01

CH-09 Sequences and Series

1. the ratio of first to the last of n A.m.'s between 5 and 35 is 1 : 4. The value of n is
   a. 9
   b. 11
   c. 10
   d. 19
2. All the terms in A.P., whose first term is a and common difference d are squared. A different series is thus formed. This series is a
   a. H.P.
   b. G.P.
   c. A.P.
   d. none of these
3. If A, G, H denote respectively the A.M., G.M. and H.M. between two unequal positive numbers, then
   a. A = GH
   b. $A^2 = GH$
   c. $A = G^2H$
   d. $G^2 = HA$
4. The next term of the sequence 1, 1, 2, 4, 7, 13,.... Is
   a. 21
   b. 24

c. none of these

d. 19

5. The sum of all non-reducible fractions with the denominator 3 lying between the numbers 5 and 8 is

a. 31

b. 52

c. 41

d. 39

6. Fill in the blanks:

The general term or n^{th} term of G.P. is given by a_n = ________.

7. Fill in the blanks:

The third term of a G.P. is 4, the product of the first five terms is ________.

8. Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+---$

9. Find the 12^{th} term of a G.P. whose 8^{th} term is 192 and the common ratio is 2.

10. Find the sum of 20 terms of an AP, whose first term is 3 and last term is 57.

11. Find the sum to n terms of the sequence: log a, log ar, log ar^2, ...

12. The n^{th} term of an AP is 4n + 1. Write down the first four terms and the 18^{th} term of an AP.

13. Find the sum to n terms in each of the series $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots$

14. Find the sum to n terms in each of the series $5^2 + 6^2 + 7^2 + ... + 20^2$

15. If a and b are the roots $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that (q + p):(q - p) = 17:15.

Solution

1. (a) 9

   Explanation:

   the sequence is $5, A_1, A_2, \ldots.. A_n, 35$
   here a=5 and b=35
   we know that, $d = \frac{b-a}{n+1} = \frac{35-5}{n+1} = \frac{30}{n+1}$
   According to question,
   $\frac{A_1}{A_n} = \frac{1}{4}$
   $\Rightarrow \frac{a+d}{a+nd} = \frac{1}{4}$
   $\Rightarrow 4a + 4d = a + nd$
   $\Rightarrow 3a = d(n-4)$
   $\Rightarrow 3 \times 5 = \left(\frac{30}{n+1}\right)(n-4)$
   $\Rightarrow (n+1) = 2(n-4)$
   $\Rightarrow n + 1 = 2n - 8$
   $\Rightarrow n = 9$

2. (d) none of these

   Explanation: the series obtained will not follow rules of AP, GP and HP

3. (d) $G^2 = HA$

   Explanation:

   Given the numbers are a and b, then we have
   $A \cdot M = A = \frac{a+b}{2}, G \cdot M = G = \sqrt{ab}$ and $H \cdot M = H = \frac{2ab}{a+b}$
   Now $AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab = (\sqrt{ab})^2 = G^2$

4. (b) 24

   Explanation:

   The given sequence can be expressed as $T_1 = T_2 = 1$
   $T_n = T_{n-1} + T_{n-2} + T_{n-3}, n \geq 3. \therefore \quad T_7 = T_6 + T_5 + T_4 = 13 + 7 + 4 = 24$

5. (d) 39

Explanation:

We have $5=\frac{15}{3}$ and $8=\frac{24}{3}$

Hence the fractions between5 and 8 with 3 as denominator will be

$\frac{16}{3},\frac{17}{3},\frac{18}{3},\frac{19}{3},\frac{20}{3},\frac{21}{3},\frac{22}{3}$ and $\frac{23}{3}$ in this only $6=\frac{18}{3}$ and $7=\frac{21}{3}$ are reducible

Now $\frac{16}{3},\frac{17}{3},\frac{18}{3},\frac{19}{3},\frac{20}{3},\frac{21}{3},\frac{22}{3}$ and $\frac{23}{3}$ is an A.P with first term a = $\frac{16}{3}$ and $d=\frac{1}{3}$

Hence Sum =$\frac{n}{2}(a+l)=\frac{8}{2}\left(\frac{16}{3}+\frac{23}{3}\right)=4(13)=52$

Hence the sum of non redubile fractions = 52-(6+7)=52-13 = 39

6. ar^{n-1}

7. 4^5

8. a = 1, r = $\frac{2}{3}$

   $S_n=\frac{a(1-r^n)}{1-r}$

   $=\frac{1\left[1-\left(\frac{2}{3}\right)^n\right]}{1-\frac{2}{3}}$

   $=3\left[1-\left(\frac{2}{3}\right)^n\right]$

   $S_5=3\left[1-\left(\frac{2}{3}\right)^5\right]=\frac{211}{81}$

9. Let a be the first term of given G.P. Here r =2 and a_8 = 192

   $\therefore a_n = ar^{n-1}$

   $\Rightarrow a_8=a\times(2)^{8-1}=192$

   $\Rightarrow a\times(2)^7=192$

   $\Rightarrow a\times 128=192$

   $\Rightarrow a=\frac{192}{128}=\frac{3}{2}$

   $\therefore a_{12} = ar^{12-1}$

   $\Rightarrow a_{12}=\frac{3}{2}\times 2^{11}=3\times 2^{10}$

   = 3 $\times$1024 = 3072

10. We have, a = 3, l = 57 and n = 20

   $\because S_n = \frac{n}{2}$ [a + l]

   $\therefore S_{20} = \frac{20}{2}$ [3 + 57]

= 10 $\times$ 60

= 600

11. We have sequence log a, log ar, log ar^2, ...

Above sequence can be expressed as log a, (log a + log r), (log a + 2 log r), ...

[$\because$ log mn = log m + log n and log n^r = r log n]

which is clearly an AP with, a = log a and d = log r

We know that, sum of n terms,

$S_n = \frac{n}{2}$ [2a + (n - 1)d]

$\therefore S_n = \frac{n}{2}$ [2 log a + (n - 1) log r]

$= \frac{n}{2}$ [log a^2 + log r^{n-1}] [$\because$x log a = log a^x]

$= \frac{n}{2}$ [log a^2r^{n-1}] [$\because$ log a + log b = log ab]

12. Here, T_n = 4n + 1 ...(i)

On putting n = 1, 2, 3, 4 in Eq. (i), we get

T_1 = 4(1) + 1 = 4 + 1 = 5

T_2 = 4 (2) + 1 = 8 + 1 = 9

T_3 = 4 (3) + 1 = 12 + 1 = 13

and T_4 = 4 (4) + 1 = 16 + 1 = 17

On putting n = 18 in Eq. (i), we get

T_{18} = 4(18) + 1 = 72 + 1 = 73

Hence, the first four terms of an AP are 5, 9, 13, 17 and 18^{th} term is 73.

13. Given: $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\ldots$ to n terms

$\therefore a_n = \frac{1}{(n^{th}\,term\;of\;1,2,3,......)(n^{th}\,term\;of\;2,3,4,.......)}$

$= \frac{1}{[1+(n-1)\times1][2+(n-1)\times1]}$

Let $\frac{1}{n(n+1)} = \frac{A}{n}+\frac{B}{n+1}$ [By partial fraction]

Then 1=A(n+1) + Bn

Put n=0 then A=1

Put n=-1 then B=-1

$\therefore \frac{1}{n(n+1)} = \frac{1}{n} + \frac{-1}{n+1}$

$\therefore a_1 = \frac{1}{1} - \frac{1}{2} a_2 = \frac{1}{2} - \frac{1}{3} a_3 = \frac{1}{3} - \frac{1}{4}$.........

And $S_n = a_1 + a_2 + a_3 + ... + a_n$

$= \frac{1}{1} - \frac{1}{n+1} = \frac{n}{n+1}$

14. Given: $5^2 + 6^2 + 7^2 + ... + 20^2$

$= (1^2 + 2^2 + 3^2 + ... + 20^2) - (1^2 + 2^2 + 3^2 + 4^2)$

$\sum_{n=1}^{20} n^2 - \sum_{n=1}^{4} n^2$

$= \frac{20(20+1)(40+1)}{6} - \frac{4(4+1)(8+1)}{6}$

$= \frac{20 \times 21 \times 41}{6} - \frac{20 \times 9}{6}$

$= \frac{20}{6}(861 - 9) = 2840$

15. Let $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$

$\therefore \frac{b}{a} = k$

$\Rightarrow$ b = ak And $\frac{c}{b} = k$

$\Rightarrow$ c = bk = (ak)k = ak^2

Also $\frac{d}{c} = k$

$\Rightarrow$ d = ck = $(ak^2)k = ak^3$

$\because$ a a and b b are the roots $x^2 - 3x + p = 0$

$\therefore a + b = \frac{-(-3)}{1} = 3$

$\Rightarrow$ a + ak = 3

$\Rightarrow$ a(1 + k) = 3 ....(i)

And $ab = \frac{p}{1}$

$\Rightarrow$ a(ak) = p

$\Rightarrow$ $a^2k = p$ ......(ii)

Also c, d are roots of $x^2 - 12x + q = 0$

$\therefore \quad c + d = \frac{-(-12)}{1} = 12$

$\Rightarrow$ $ak^2 + ak^3 = 12$

$\Rightarrow$ $ak^2(1 + k) = 12$ ....(iii)

And $cd = \frac{q}{1}$

$\Rightarrow$ $ak^2(ak^3) = q$

$\Rightarrow$ $a^2k^5 = q$ ....(iv)

Dividing eq. (iii) by eq. (i), $\frac{ak^2(1+k)}{a(1+k)} = \frac{12}{3}$

$\Rightarrow$ $k^2 = 4$

$\Rightarrow k = \pm 2$

Now $\frac{q+p}{q-p} = \frac{a^2k^5+a^2k}{a^2k^5-a^2k} = \frac{a^2k(k^4+1)}{a^2k(k^4-1)}$

$= \frac{(\pm 2)^4+1}{(\pm 2)^4-1} = \frac{16+1}{16-1} = \frac{17}{15}$

Therefore, (q + p):(q - p) = 17:15