CBSE Test Paper 02
CH-08 Binomial Theorem

1. The greatest term in the expansion of $(3+2x)^9$,when x = 1, is
   a. 5^{th}
   b. 7^{th}
   c. 4^{th}
   d. 6^{th}
2. The index of the power of x that occurs in the 7^{th} term from the end in the expansion of $\left(\frac{2x}{3}-\frac{3}{2x}\right)^9$ is
   a. -5
   b. 3
   c. -3
   d. 5
3. $(\sqrt{3}+1)^{2n}+(\sqrt{3}-1)^{2n}$is
   a. negative real number
   b. an even positive integer
   c. an odd positive integer
   d. irrational number
4. In the expansion of $(1+x)^{60}$, the sum of coefficients of odd powers of x is
   a. 2^{58}
   b. 2^{60}
   c. 2^{61}
   d. 2^{59}
5. The index of the power of x that occurs in the 6^{th} term in the expansion of $\left(\frac{4x}{5}-\frac{8}{5x}\right)^9$ is
   a. 2
   b. 1
   c. 3
   d. -1
6. Fill in the blanks:

The coefficients of 2nd, 3rd, and 4th terms in the binomial expansion of $(1 + x)^n$ are ${}^nC_1, {}^nC_2$ and ${}^nC_3$, respectively.

7. Fill in the blanks:

   The number of terms in the binomial expansion of $(1+\sqrt{5}x)^7 + (1-\sqrt{5}x)^7$ is ________.

8. Write the general term in the expansion of $(x^2 - y)^6$
9. Using binomial theorem, write down the expansion: $\left(x - \frac{1}{x}\right)^6$
10. Write the general term in the expansion of $(x^2 - yx)^{12}$
11. Find the number of terms in the expansion of $(1 + 2x + x^2)^{14}$
12. Find the term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$
13. Find the 4 th term in the expansion of $(x - 2y)^{12}$.
14. Find the coefficients of : x^5 in $(x + 3)^8$
15. If O be the sum of odd terms and E that of even terms in the expansion of $(x + a)^n$, prove that:

    i. $O^2 - E^2 = (x^2 - a^2)^n$
    ii. $4\,OE = (x + a)^{2n} - (x - a)^{2n}$
    iii. $2\,(O^2 + E^2) = (x + a)^{2n} + (x - a)^{2n}$

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Solution

1. (a) 5^{th}

   Explanation:

   We have the general term in the expansion of $(3+2x)^9$ is given by
   $T_{r+1} = ^9C_r \quad (3)^{9-r}(2x)^r$
   Now $\frac{T_{r+1}}{T_r} = \frac{^9C_r \quad (3)^{9-r}(2x)^r}{^9C_{r-1} \quad (3)^{10-r}(2x)^{r-1}}$
   $= \frac{9!(3)^{9-r}(2x)^r}{(9-r)!\cdot r!} \times \frac{(10-r)!(r-1)!}{9!(3)^{10-r}(2x)^{r-1}}$
   $= \frac{(10-r)\cdot 2x}{3r}$
   $= \frac{2(10-r)}{3r}$ since given x=1
   Now $\frac{T_{r+1}}{T_r} \geq 1$
   $\Rightarrow \frac{(20-2r)}{3r} \geq 1$
   $\Rightarrow 20-2r \geq 3r$
   $\Rightarrow r \leq \frac{20}{5}$
   $\Rightarrow r \leq 4$
   Hence the maximum value of r is 4
   Now $T_5 = T_{4+1} = ^9C_4 \quad (3)^{9-4}(2x)^4 = 126 \times 243 \times 16(x)^4$=489888
2. (c) -3

   Explanation:
   We have the general term of $(x+a)^n$ is $T_{r+1} = ^nC_r \quad (x)^{n-r}a^r$
   Now consider $\left(\frac{2x}{3} - \frac{3}{2x}\right)^9$
   Here $T_{r+1} = ^9C_r \quad \left(\frac{2x}{3}\right)^{9-r}\left(-\frac{3}{2x}\right)^r$
   We have 7 th term from the end=(12-7+2) t h term from the beginning
   Required term is $T_7 = T_{6+1} = ^9C_6 \quad \left(\frac{2x}{3}\right)^{9-6}\left(-\frac{3}{2x}\right)^6 = ^9C_6 \quad \times$
   $(2x)^{3-6}3^{6-3} = ^9C_6 \quad \left(\frac{3}{2}\right)^3 x^{-3}$

3. (b) an even positive integer

Explanation:

We have $(a+b)^n+(a-b)^n$

$= \left[{}^nC_0 a^n +{}^n C_1 \quad a^{n-1}b +{}^n C_2 \quad a^{n-2}b^2 +{}^n C_3 \quad a^{n-3}b^3 + \ldots\ldots\ldots\ldots +{}^n C_n\right.$ $\left[{}^nC_0 \quad a^n -{}^n C_1 a^{n-1}b +{}^n C_2 \quad a^{n-2}b^2 -{}^n C_3 \quad a^{n-3}b^3 + \ldots\ldots + (-1)^n \cdot{}^n C_n\right]^n$

$= 2\left[{}^nC_0 \quad a^n +{}^n C_2 \quad a^{n-2}b^2 + \ldots\ldots\ldots\ldots\ldots\right]$

Let $a=\sqrt{3}$ and b=1 and n=2n

$(\sqrt{3}+1)^{2n}+(\sqrt{3}-1)^{2n} = 2\left[{}^{2n}C_0(\sqrt{3})^{2n} +{}^{2n} C_2(\sqrt{3})^{2n-2} 1^2 +{}^{2n} C_4(\sqrt{3})^{2n-4} 1\right.$

$= 2\left[{}^{2n}C_0 \quad (3)^n +{}^{2n} C_2 \quad (3)^{n-1} +{}^{2n} C_4 \quad (3)^{n-2} + \ldots\ldots\ldots\ldots\ldots\right]$

=2(a positive integer)

Hence we have $(\sqrt{3}+1)^{2n}+(\sqrt{3}-1)^{2n}$} is always an even positive integer

4. (d) 2^{59}

Explanation:

We have

$$(1+x)^n ={}^n C_0 \quad +{}^n C_1 \quad (x) +{}^n C_2 \quad (x)^2 + \ldots\ldots\ldots\ldots + (x)^n \quad \ldots\ldots(i)$$

Also

$$(1-x)^n ={}^n C_0 \quad -{}^n C_1 \quad (x) +{}^n C_2 \quad (x)^2 - \ldots\ldots\ldots\ldots + (-1)^n (x)^n \quad \ldots\ldots(ii)$$

Now

$(1-x)^n = \left[{}^nC_0 +{}^n C_2(x)^2 +{}^n C_4(x)^4 + \ldots\ldots\right] - \left[{}^nC_1 \quad (x) +{}^n C_3(x)^3 +{}^n C_5(x\right.$

Let x=1 and n=60

From equation (i), we get

$2^{60} ={}^{60} C_0 \quad +{}^{60} C_1 \quad +{}^{60} C_2 \quad + \ldots\ldots\ldots\ldots +{}^{60}C_{60}$

$\Rightarrow 2^{60}$ =(sum of coefficients of even powers of x) + (sum of coefficients of odd powers of x) .....(iii)

From equation (ii), we get

$0 = \left[{}^{60}C_0 +{}^{60} C_2 +{}^{60} C_4 + \ldots\ldots\ldots\right] - \left[{}^{60}C_1 +{}^{60} C_3 +{}^{60} C_5 + \ldots\ldots\ldots\ldots\right]$

$\Rightarrow 0$ =(sum\quad of coefficients of even powers of x) + (sum of coefficients of odd powers of x) .....(iii)

Now subtract equation (iv) from equation (iii), we get

$2^{60} - 0 = 2$ (sum of coefficients of odd powers of x)

⇒(sum of coefficients of odd powers of x $x = 2^{59}$)

5. (d) -1

   Explanation:

   We have the general term of

   $(x+a)^n \quad is \quad T_{r+1} = ^n C_r \quad (x)^{n-r} a^r$

   Now consider $\left(\frac{4x}{5} - \frac{8}{5x}\right)^9$

   Here$T_{r+1} = ^9 C_r \quad \left(\frac{4x}{5}\right)^{9-r} \left(-\frac{8}{5x}\right)^r$

   Required\quad term\quad is

   $$T_6 = \quad T_{5+1} = \quad {}^9C_5 \quad \left(\frac{4x}{5}\right)^{9-5} \left(-\frac{8}{5x}\right)^5 = -{}^9C_5 \quad \times \left(\frac{4}{5}\right)^4 \times \left(\frac{8}{5}\right)^5 x^{4-5}$$
   $$= -{}^9C_5 \quad \times \left(\frac{4}{5}\right)^4 \times \left(\frac{8}{5}\right)^5 \times x^{-1}$$

   Hence the power of x=-1

6. True

7. 4

8. Here general term in the expansion of $(x^2 - y)^6$ is

   $T_{r+1} = {}^6C_r (x^2)^{6-r} (-y)^r$
   $= (-1)^r {}^6C_r x^{12-2r} y^r$

9. $x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$

10. Here general term in the expansion of $(x^2 - yx)^{12}$ is

    $T_{r+1} = {}^{12}C_r (x^2)^{12-r} (-yx)^r$
    $= (-1)^r {}^{12}C_r x^{24-2r} \cdot y^r x^r = (-1)^r {}^{12}C_r x^{24-r} \cdot y^r$

11. Here $(1 + 2x + x^2)^{14} = [(1 + x)^2]^{14} = (1 + x)^{28}$

    The number of terms in the expansion of $(1 + 2x + x^2)^{14}$ = 28 + 1=29

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12. The general term of $\left(3x - \frac{2}{x^2}\right)^{15}$ is

$T_{r+1} = {}^{15}C_r (3x)^{15-r} \left(\frac{-2}{x^2}\right)^r$

$= {}^{15}C_r (3)^{15-r} (-2)^r x^{15-3r}$

For term independent term of x, put 15 - 3r = 0 ⇒ r = 5

∴ The term independent of x = ${}^{15}C_5 (3)^{15-5} (-2)^5 = -3003 (3^{10}) (2^5)$

13. Here general term in the expansion of $(x - 2y)^{12}$ is

$T_{r+1} = {}^{12}C_r (x)^{12-r} \cdot (-2y)^r$

$= (-1)^r {}^{12}C_r 2^r \cdot x^{12-r} \cdot y^r$

Putting r = 3

$\therefore T_3 = (-1)^3 {}^{12}C_3 2^3 x^{12-3} \cdot y^3 = -{}^{12}C_3 \cdot 8x^9 y^3$

$= -220 \times 8x^9 y^3 = -1760x^9 y^3$

14. Here general term of the expansion $(x + 3)^8$ is

$T_{r+1} = {}^8C_r (x)^{8-r} (3)^r$ ....(1)

Now 8 - r = 5 ⇒ r = 8 - 5 = 3 . . .

∴ Putting r = 3 in (1)

$T_4 = {}^8C_3 x^5 (3)^3$

Coefficient of x^5 on the expansion $(x + 3)^8 = {}^8C_3 \cdot (3)^3 = 1512$

15. We have,

$(x + a)^n = \{{}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \ldots + {}^nC_{n-1} x a^{n-1} + {}^nC_n a^n$

$\Rightarrow (x + a)^n = \{{}^nC_0 x^n a^0 + {}^nC_2 x^{n-2} a^2 + \ldots\} + \{{}^nC_1 x^{n-1} a^1 + {}^nC_3 x^{n-3} a^{3} + \ldots\}$

$\Rightarrow (x + a)^n = O + E$ ...(a)

and, $(x - a)^n = {}^nC_0 x^n - {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 - {}^nC_3 x^{n-3} a^3 + \ldots + {}^nC_{n-1} x (-1)^{n-1} a^{n-1} + {}^nC_n (-1)^n a^n$

$\Rightarrow (x - a)^n = \{{}^nC_0 x^n + {}^nC_2 x^{n-2} a^2 + \ldots\} - \{{}^nC_1 x^{n-1} a^1 + {}^nC_3 x^{n-3} a^{3} + \ldots\}$

$\Rightarrow (x - a)^n = O - E$ ... (b)

i. Multiplying (a) and (b), we get

$(x + a)^n \ (x - a)^n = (O + E)(O - E)$

$\Rightarrow (x^2 - a^2)^n = O^2 - E^2$

ii. We have,

$4\,OE = (O+E)^2 - (O - E)^2$

$\Rightarrow 4\,OE = \{((x + a)^n\}^2 - \{(x - a)^n\}^2$ [Using (a) and (b)]

$\Rightarrow 4\,OE = (x + a)^{2n} - (x - a)^{2n}$

iii. Squaring (a) and (b) and then adding, we get

$(x + a)^{2n} + (x - a)^{2n} = (O + E)^2 + (O - E)^2 = 2(O^2 + E^2)$.

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