CBSE Test Paper 01

CH-08 Binomial Theorem

1. Find the coefficient of x^6y^3 in the expansion of $(x+2y)^9$
   a. 1008
   b. 336
   c. 84
   d. 672
2. The coefficients of x^p and x^q (p, q are + ve integers) in the binomial expansion of $(1+x)^{p\ +\ q}$ are
   a. reciprocal to each other
   b. unequal
   c. additive inverse of each other
   d. equal
3. $(\sqrt{5}+1)^{2n+1}-(\sqrt{5}-1)^{2n+1}$is
   a. 0
   b. an even positive integer
   c. an odd positive integer
   d. not an integer
4. The term independent of x in the expansion of $\left(x-\frac{3}{x^2}\right)^{18}$ is
   a. 3^6
   b. ${}^{18}C_6\ 3^6$

c. ${}^{18}C_6$

d. ${}^{18}C_{12}$

5. The 1st three terms in the expansion of $(2 + \frac{x}{3})^4$ are

a. $16 + \frac{32x}{3} + \frac{24x^2}{9}$

b. $16 + \frac{34x}{3} + \frac{24x^2}{9}$

c. $16 + 12x + \frac{3}{16}x^2$

d. $16 + 3x - \frac{3}{16}x^2$

6. Fill in the blanks:

The value of ${}^8C_5$ is _______.

7. Fill in the blanks:

In the binomial expansion of $(a + b)^9$, the middle terms is _______ term.

8. Find the value of ${}^8C_5$.

9. Find the number of terms in expansions of $(2x - 3y)^9$.

10. Using binomial theorem, write down the expansion: $(2x + 3y)^5$.

11. Prove that $\sum_{r=0}^{n} 3^r {}^nC_r = 4^n$

12. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$.

13. Expand the given expression $\left(\frac{2}{x} - \frac{x}{2}\right)^5$

14. If the third term in the expansion of $\left(\frac{1}{x} + x^{\log_{10} x}\right)^5$ is 1000, then find x.

15. Find the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$ and find the relation between a and b so that these coefficients are equal.

Solution

1. (d) 672

 Explanation:

 We have the general term of $(x+a)^n$ is $T_{r+1}={}^n C_r \quad (x)^{n-r}a^r$

 Now consider $(x+2y)^9$

 Here $T_{r+1}={}^9 C_r \quad (x)^{9-r}(2y)^r={}^9 C_r \quad (2)^r(x)^{9-r}(y)^r$

 Comparing the indices of x as well as y in x^6y^3 and in T_{r+1} , we get

 $(x)^{9-r}(y)^{r^2}=x^6y^3$

 $\Rightarrow r=3$

 $\therefore \quad T_4=T_{3+1}={}^9 C_3 \quad (x)^{9-3}(2y)^3$

 Hence coefficient of $x^6y^3=8.{}^9 C_3=672$

2. (d) equal

 Explanation:

 We have the general term of $(x+a)^n$ is $T_{r+1}={}^n C_r \quad (x)^{n-r}a^r$

 Now consider $(1+x)^{p+q}$

 Here $T_{r+1}={}^{p+q} C_r \quad (1)^{(p+q)-r}(x)^r$

 Comparing the indices of x in x^p and in T_{r+1},we get p=r

 Therefore the coefficient of x^p is $T_{p+1}={}^{p+q} C_p$

 Now again comparing the indices of x in x^q and in T_{r+1} ,we get q=r

 Therefore the coefficient of x^q is $T_{p+1}={}^{p+q} C_p$

 But we have ${}^{p+q}C_p={}^{p+q} C_q \ [\because {}^n C_r={}^n C_{n-r}]$

3. (b) an even positive integer

 Explanation:

 We have $(a+b)^n-(a-b)^n$

 $=[nC_0 \quad a^n+{}^n C_1 \quad a^{n-1}b+{}^n \quad C_2 \quad a^{n-2}b^2+{}^n C_3 \quad a^{n-3}b^3+\ldots\ldots\ldots+{}^n C_n$

 $-\left[{}^n C_0 \quad a^n-{}^n C_1 \quad a^{n-1}b+{}^n C_2 \quad a^{n-2}b^2-{}^n C_3 \quad a^{n-3}b^3+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\right.$

 $=2\left[{}^n C_1 \quad a^{n-1}b+{}^n C_3 \quad a^{n-3}b^3+\ldots\ldots\ldots\ldots\ldots\right]$

 $Let \quad a=\sqrt{5} \quad and \quad b=1 \quad and \quad n=2n+1$

 Now we get $(\sqrt{5}+1)^{2n+1}-(\sqrt{5}-1)^{2n+1}$

$= 2\left[{}^{2n+1}C_1(\sqrt{3})^{2n} + {}^{2n+1}C_3(\sqrt{3})^{2n-2}1^3 + {}^{2n+1}C_5(\sqrt{3})^{2n-4}1^5 + \ldots\right]$

$= 2\left[{}^{2n+1}C_1 \quad (3)^n + {}^{2n+1}C_3 \quad (3)^{n-1} + {}^{2n+1}C_5 \quad (3)^{n-2} + \ldots\ldots\ldots\ldots\ldots\right]$

=2(a positive integer)

Hence we have $(\sqrt{5}+1)^{2n} - (\sqrt{5}-1)^{2n+1}$ is an even positive integer

4. (b) ${}^{18}C_6 \; 3^6$

Explanation:

We have the general term of $(x+a)^n$ is $T_{r+1} = {}^nC_r \quad (x)^{n-r}a^r$

Now consider $\left(x - \frac{3}{x^2}\right)^{18}$

Here $T_{r+1} = {}^{18}C_r \quad (x)^{18-r}\left(-\frac{3}{x^2}\right)^r$

The term independent of x means index of x is 0.

Comparing the indices of x in x^0 and in T_{r+1} , we get

$18 - r - 2r = 0 \Rightarrow r = \frac{18}{3} = 6$

Therefore the required term is $T_{6+1} = {}^{18}C_6$

$(x)^{18-6}\left(-\frac{3}{x^2}\right)^6 = {}^{18}C_6.3^6$

5. (a) $16 + \frac{32x}{3} + \frac{24x^2}{9}$

Explanation:

We have $(x+a)^n = {}^nC_0 \quad x^n + {}^nC_1 \quad x^{n-1}a + {}^nC_2 \quad x^{n-2}a^2 + {}^nC_3 \quad x^{n-3}a^3 + \ldots\ldots\ldots + C_n \quad a^n$

Now consider $\left(2 + \frac{x}{3}\right)^4$

Here $x = 2, a = \frac{x}{3}, n = 4$

$\left(2+\frac{x}{3}\right)^4 = {}^4C_0 \quad (2)^4 + {}^4C_1 \quad (2)^3\left(\frac{x}{3}\right) + {}^4C_2 \quad (2)^2\left(\frac{x}{3}\right)^2 + {}^4C_3 \quad (2)^1\left(\frac{x}{3}\right)^3 +$

$= 16 + \frac{32x}{3} + \frac{24x^2}{9} + \ldots\ldots\ldots$

6. 56

7. 5th and 6th

8. ${}^8C_5 = \frac{8!}{(8-5)!5!}$ [$\because {}^nC_r = \frac{n!}{(n-r)!r!}$]

$= \frac{8!}{3!5!} = \frac{8\times7\times6\times5\times4\times3\times2\times1}{3\times2\times1\times5\times4\times3\times2\times1} = 56$

9. The expansion of $(x + a)^n$ has $(n + 1)$ terms. So, the expansion of $(2x - 3y)^9$ has $(9 + 1)$ = 10 terms.

10. Using formula, $(a+b)^n=\sum_{k=0}^n {}^nC_k(a^{n-k}b^k)$

    $(2x + 3y)^5 = 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

11. $\sum_{r=0}^n \cdot C_r a^{n-r}b^r=(a+b)^n.......(1)$

    Now, $\sum_{r=0}^n 3^{rn}C_r=\sum_{r=0}^n nC_r\,(1)^{n-r}.3^r=(1+3)^n$=$(1+3)^n$ (by (1)

    =4

12. We have, $(1 - 3x + 7x^2)(1 - x)^{16}$

    $= (1 - 3x + 7x^2)(^{16}C_0 - {}^{16}C_1x + {}^{16}C_2x^2 + ...)$

    $= (^{16}C_0 - {}^{16}C_1x + {}^{16}C_2x^{2 - ...}) - 3x(^{16}C_0 - {}^{16}C_1x + ...) + 7x^2(^{16}C_0 - {}^{16}C_1x + ...)$

    Here, the term containing x is $-{}^{16}C_1x - 3\,{}^{16}C_0x = -16x - 3x$

    ∴ Coefficient of x = - 16 - 3 = - 19

13. Using binomial theorem for the expansion of $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ we have

    $\left(\frac{2}{x}-\frac{x}{2}\right)^5={}^5C_0\left(\frac{2}{x}\right)^5+{}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{-x}{2}\right)+{}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{-x}{2}\right)^2+{}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{-x}{2}\right)^3$
    $+{}^5C_4\left(\frac{2}{x}\right)\left(\frac{-x}{2}\right)^4+{}^5C_5\left(\frac{-x}{2}\right)^5$
    $=\frac{32}{x^5}+5\cdot\frac{16}{x^4}\cdot\frac{-x}{2}+10\cdot\frac{8}{x^3}\cdot\frac{x^2}{4}+10\cdot\frac{4}{x^2}\cdot\frac{-x^3}{8}+5\cdot\frac{2}{x}\cdot\frac{x^4}{16}+\frac{-x^5}{32}$
    $=\frac{32}{x^5}-\frac{40}{x^3}+\frac{20}{x}-5x+\frac{5}{8}x^3-\frac{x^5}{32}$

14. We have,

    $T_3 = 1000$

    $\Rightarrow T_{2+1} = 1000$

    $\Rightarrow {}^5C_2\left(\frac{1}{x}\right)^{5-2}\left(x^{\log_{10}x}\right)^2 = 1000$

    $\Rightarrow 10\left(x^{\log_{10}x}\right)^2\times x^{-3} = 1000$

    $\Rightarrow x^{2\log_{10}x}\times x^{-3} = 100$

    $\Rightarrow x^{2\log_{10}x-3} = 10^2$

    $\Rightarrow 2\log_{10}x-3 = \log_x 10^2$ [taking $\log_x$ both sides]

    $\Rightarrow 2\log_{10}x-3 = 2\log_x 10$

    $\Rightarrow 2\log_{10}x-3 = \frac{2}{\log_{10}x}$ [using $\log_a b = \frac{1}{\log_b a}$]

$\Rightarrow 2y - 3 = \frac{2}{y}$, where y = $\log_{10}x$

$\Rightarrow 2y^2 - 3y - 2 = 0$

$\Rightarrow (2y + 1)(y - 2) = 0$

$\Rightarrow y = 2$ or $y = -\frac{1}{2}$

$\Rightarrow \log_{10}x = 2$ or $\log_{10}x = -\frac{1}{2} \Rightarrow x = 10^2 = 100$ or $x = 10^{-\frac{1}{2}}$

15. Suppose x^7 occurs in $(r + 1)^{th}$ term of the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$

Now,

$T_{r+1} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r = {}^{11}C_r\, a^{11-r}\, b^{-r}\, x^{22-3r}$ ...(i)

This will contain x^7, if

$22 - 3r = 7 \Rightarrow 3r = 15 \Rightarrow r = 5.$

Putting r = 5 in (i), we obtain that

Coefficient of x^7 in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is ${}^{11}C_5\, a^6\, b^{-5}$

Suppose x^{-7} occurs in $(r + 1)^{th}$ term of the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$

Now,

$T_{r+1} = {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2}\right)^r = {}^{11}C_r\, a^{11-r} (-1)^r\, b^{-r}\, x^{11-3r}$ ...(ii)

This will contain x^{-7}, if

$11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

Putting r = 6 in (ii), we obtain that

Coefficient of x^{-7} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$ is ${}^{11}C_6\, a^5\, b^{-6} (-1)^6$

If the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$, then

${}^{11}C_5\, a^6\, b^{-5} = {}^{11}C_6\, a^5\, b^{-6} (-1)^6 \Rightarrow {}^{11}C_5\, ab = {}^{11}C_6 \Rightarrow ab = 1$ $[\because {}^{11}C_5 = {}^{11}C_6]$