**CBSE Test Paper 02**

**CH-07 Permutations & Combinations**

1. A convex polygon of n sides has n diagonals. Then value of n is
   a. 6
   b. 8
   c. 7
   d. 5
2. The total number of 4 digit odd numbers that can be formed using 0, 1, 2, 3, 5, and 7 are
   a. 375
   b. 720
   c. 400
   d. 520
3. The number of all even divisors of 1600 is
   a. none of these
   b. 21
   c. 18
   d. 3
4. The number of all odd divisors of 3600 is
   a. 9
   b. 18
   c. none of these
   d. 45
5. The number of all three digit even numbers such that if 5 is one of the digits then next digit is 7 is
   a. 370
   b. 360

c. none of these

d. 365

6. Fill in the blanks:

   If n and r are positive integers such that $1 \leq r \leq n$, then the number of permutations of n distinct things taken r at a time is denoted by ________ .

7. Fill in the blanks:

   If the letters of the word RACHIT are arranged in all possible ways as listed in the dictionary, then the rank of the word RACHIT is ________

8. Find the number of different 4-digit numbers that can be formed with the digits 2, 3, 4, 7 and using each digit only once.

9. If ${}^{n}C_{10} = {}^{n}C_{12}$, then find the value of ${}^{23}C_{n}$.

10. Evaluate $2 \times 6! - 3 \times 5!$

11. Find the sum of n terms of the series $1 \times 2, 2 \times 3, 3 \times 4, 4 \times 5, \ldots$

12. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of $2^{nd}$ hour, $4^{th}$ hour and $n^{th}$ hour?

13. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

14. How many words, with or without meaning can be made from the letters of the word, MONDAY, assuming that no letter is repeated if
    (i) 4 letters are used at a time
    (ii) all letters are used at a time
    (iii) all letters are used but first letter is a vowel?

15. How many three-digit numbers are there, with no digit repeated?

## Solution

1. (d) 5

   **Explanation:** We have an n sided polygon has n vertices.

   If you join every distinct pair of vertices you will get ${}^nC_2$ lines.

   These ${}^nC_2$ lines account for the n sides of the polygon as well as for the diagonals.

   So the number of diagonals is given by ${}^nC_2 - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$

   But given number of sides = number of diagonals =n

   $\Rightarrow n = \frac{n(n-3)}{2}$
   $\Rightarrow 2n = n(n-3)$
   $\Rightarrow n - 3 = 2$
   $\Rightarrow n = 5$

2. (b) 720 Explanation:

   We have to find the total number of four digit odd numbers formed using the digits 0,1,2,3,5,7

   Since it is an odd number the last place ( unit's place) can be filled by any of the odd numbers 1,3,5,7 in 4 different ways.

   Since repetition is allowed the second and third places can be filled by any of the six given digits

   Since it has to be a four digit number the first place can be filled by any of the five given digits other than zero in 5 ways

   Hence all the four places can be filled in $4 \times 6 \times 6 \times 5 = 720$ ways

3. (c) 18 Explanation:

   We have 1600=$2^6 .5^2$

To form factors we have to do selections from a lot of 2's and 5's and multiply them together.

To form even factor we should choose at least one 2's from the lot , which will ensure that what ever be the remaining selection, their multiplication will always result in an even factor.

The number of ways to select atleast one ‘2’ from a lot of six identical ‘2’s will be 6 (i.e. select 1 or select 2 or select 3 or select 4 or select 5 or select 6)

And, we’ll select any number of ‘5 from a lot of two identical ‘5’s in 3 ways(select 0, select 1,select 2)

There fore the total number of selection of even factors=6x3=18

4. (a) 9 Explanation:

$we \quad have \quad 3600 = 2^4 . 3^2 . 5^2$

To get the odd factors we will get rid of 2's

We will make the selection from only 3's and 5's

Number of ways 3 can be selected from a lot of two 3's= 3 ways ( one 3,two 3's or three 3's)

Number of ways 5 can be selected from a lot of two 5's= 3 ways ( one 5,two 5's or three 5's)

Therefore the number of odd factors is 3600= 3 X 3 =9

5. (d) 365
**Explanation:**

You have two different kinds of such three-digit even numbers. First is 5 at the hundred's place and second 5 is not at the hundred's place

So total we have 360 + 5 = 365 possibilities.

- In first case no is of the form 57x, where x is the unit's digit ,which can be 0,2,4,6,8 which is just 5 possibilities.Hence the no of possibilities in this case is 1x1x5=5
- In second case the hundred's digit can be 1,2,3,,4,,6,7,8,or,9 which is 8 ways and the ten's digit can be any of the 9 numbers and unit digit can be any of the 5 even numbers .Therfore the no: of ways will be 8×9×5=360

6. $^nP_r$

7. 481

8. Given, total number of digits are 4.

   $\therefore$ Total 4-digit numbers can be formed = $^4P_4$ = 4! = 24

9. We know that, $^nC_x = {}^nC_y \Leftrightarrow x + y = n$ or $x = y$

   Here $x \neq y$, so $x + y = n$.
   $\Rightarrow n = 10 + 12 = 22$

   Now, $^{23}C_n = {}^{23}C_{22} = 23$

10. We have,
    $2 \times 6! - 3 \times 5! = 2 \times 6 \times 5! - 3 \times 5!$
    = 5! (12 - 3)
    = 5! $\times$ 9
    $= 5 \times 4 \times 3 \times 2 \times 1 \times 9$
    = 1080

11. Given series: S = 1 $\times$ 2, 2 $\times$ 3, 3 $\times$ 4, 4 $\times$ 5, ...

    Then, $n^{th}$ term, $T_n = n(n + 1) = n^2 + n$

    $\therefore T_n = n^2 + n$

    On taking summation from 1 to n on both sides, we get
    $S_n = \sum_1^n T_n = \sum_1^n n^2 + \sum_1^n n$
    = $\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$
    = $\frac{n(n+1)}{6}[2n + 1 + 3]$

= $\frac{n(n+1)}{6}[2n+4]=\frac{n(n+1)(n+2)}{3}$

12. Bacteria present in the culture originally = 30

    Since the bacteria doubles itself after each hour, then the sequence of bacteria after each hour is a G.P.

    Here a = 30 and r = 2

    $\therefore$ Bacteria at the end of 2nd hour = $30\times2^{3-1}=30\times2^2=120$

    And Bacteria at the end of 4th hour = $30\times2^{5-1}=30\times2^4=480$

    And Bacteria at the end of nth hour =$a_{n+1}=30(2^{(n+1)-1})=30(2^n)$

13. Total letters of the word MISSISSIPPI = 11

    Here M=1, I = 4, S = 4 and P = 2

    $\therefore$ Number of permutations $=\frac{11!}{4!4!2!}$

    $=\frac{11\times10\times9\times8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1\times2\times1}=34650$

    when the four I's come together then it becomes one letter so total number of letters in the word when all I's come together = 8.

    $\therefore$ Number of Permutations $=\frac{8!}{4!2!}=\frac{8\times7\times6\times5\times4!}{4!\times2\times1}=840$

    Number of permutations when four I's do not come together = 34650 - 840 = 33810.

14. Total number of letters in word MONDAY = 6

    Number of vowels in word MONDAY = 2

    (i) Number of letters used = 4

    $\therefore$ Number of permutations $={}^6P_4=\frac{6!}{(6-4)!}$

    $=\frac{6!}{2!}=\frac{6\times5\times4\times3\times2!}{2!}=360$

    (ii) Number of letters used = 6

    $\therefore$ Number of permutations $={}^6P_6$

    $=\frac{6!}{0!}=6\times5\times4\times3\times2\times1=720$

    (iii) Here the first letter is vowel

    $\therefore$ Number of permutation of vowel $={}^2P_1=\frac{2!}{1!}=2$

    Now the remaining five places can be filled with remaining five letters.

    $\therefore$ Number of permutations $={}^5P_5=\frac{5!}{0!}=5\times4\times3\times2\times1=120$

Thus total number of permutations $= 2 \times 120 = 240$

15. Total number of digits = 10

Total number of 3 digit number = ${}^{10}P_3$

But these arrangments also include those numbers which have 0 at hundred's place. Such number are not 3-digits numbers.
When 0 is fixed at hundred's place, we have to arrange remaining 9 digits by taking 2 at a time.

The number of such arrangments is ${}^{9}P_3$.

So, the total of numbers having 0 at hundred's place = ${}^{9}P_2$

Hence, total number of 3 digits numbers which distinct = ${}^{10}P_3$-${}^{9}P_2$

$= \frac{10!}{(10-3)!} - \frac{9!}{(9-2)!}$

$= \frac{10!}{7!} - \frac{9!}{7!}$

$= \frac{10\times9\times8\times7!}{7!} - \frac{9\times8\times7!}{7!}$

= 720 - 72

= 648