CBSE Test Paper 01
CH-07 Permutations & Combinations

1. The greatest possible number of points of intersection of 8 straight lines and 4 circles is
   a. 32
   b. 104
   c. 128
   d. 64
2. Number of ways in which 10 different things can be divided into two groups containing 6 and 4 things respectively is
   a. P(10,4)
   b. P(10,2)
   c. P(10,6)
   d. C(10 , 4)
3. The number of arrangements of n different things taken r at a time which include a particular thing is
   a. P(n-1, r-1)
   b. none of these
   c. n P(n-1,r)
   d. r P (n-1,r-1)
4. The number of all selections which a student can make for answering one or more questions out of 8 given questions in a paper, when each question has an alternative, is:
   a. 255
   b. 6561

c. 6560
d. 256

5. The number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated is

a. $5!.4!$

b. none of these

c. $4!.4!$

d. 288

6. Fill in the blanks:

The continued product of first n natural numbers, is called the ________.

7. Fill in the blanks:

6 different rings can be worn on the four fingers of hand in ________ ways.

8. If there are six periods in each working day of a school, then in how many ways we can arrange 5 subjects such that each subject is allowed at least one period?

9. Compute $\frac{8!}{6!\times 2!}$

10. Compute. $\frac{8!}{4!}$, is $\frac{8!}{4!} = 2!$?

11. How many 5-digit telephone numbers can be constructed using the digits 0 to 9, if each number starts with 67 (e.g., 67125 etc.) and no digit appears more than once?

12. Find the values of the following:

i. $^5P_3$
ii. P (15, 3)

13. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for the assistance of which at least 18 families must have at most 2 children. In how many ways, can the

choice be made?

14. In how many ways, can the letters of the word 'HONESTY' be arranged? Do you like jumbled letters of word HONESTY? Why honesty is acquired in your life?

15. If P (15, r -1) : P (16, r - 2) = 3:4, find r.

Solution

1. (b) 104 Explanation:
   Every two straight lines can make one point of intersection.
   Number of points of intersection=$^8C_2 \quad .1 = 28$
   Every two circles can make two points of intersection.
   Number of points of intersection $=^4 C_2 \quad .2 = 12$
   Each circle can make two intersection points with each straight line
   Number of points of intersection $=^4 C_1 \quad .^8 C_1 \quad .2 = 64$
   Therefore, required number of points of intersection
   =28+12+64=104
2. (d) C(10 , 4)
   Explanation:

   If there are 10 things and we have to make them in to two groups containing 6 things and 4 things respectively , you have to select 6 to form first group , then automatically another group would have formed of 4 remaining things.

   Now 6 things can be selected from 10 things in $^{10}C_6$ different ways

   Also we have $^{10}C_6 \quad =^{10} C_4 \quad [\because ^n C_r \quad =^n C_{n-r} \quad]$

3. (d) r P (n-1,r-1)
   Explanation: The number of arrangements of n different things taken r at a time which include a particular thing is r $^{n-1}P_{r-1}$ = rP(n-1,r-1)
4. (c) 6560 Explanation:

   Since a student can solve every question in three ways- either he can attempt the first alternative , or the second alternative or he does not attemp that question

   Hence the total ways in which a sudent can attempt one or more of 8 questions =3^8

   Therefore to find the number of all selections which a student can make for answering one or more questions ou tof 8 given questions =$3^8 - 1 = 6560$ [we will have to exclude only the case of not answering all the 8 questions]

5. (d) 288

   Explanation:

   4 flowers which are always together can be considered as one SET, Therefore we have to arrange one SET (4 flowers) and 4 other flowers into a garland. Which means, 5 things to be arranged in a garland.

   (5-1)!

   And the SET of flowers can arrange themselves within each other in 4! ways.

   Therefore (5-1)!*(4!)

   But, Garland, looked from front or behind does not matter. Therefore the clockwise and anti clockwise observation does not make difference.

   Therefore

   (5-1)! * (4!)/ 2 = 288.

6. 'n factorial'

7. $(4)^6$

8. Six periods can be arranged for 5 subjects in ${}^6P_5$ ways

   $= \frac{6!}{1!} = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ = 720

   One period is left, which can be arranged for any of the 5 subjects. One left period can be arranged in 5 ways.

   $\therefore$ Required number of arrangements $= 720 \times 5$ = 3600

9. $\frac{8!}{6! \times 2!} = \frac{8 \times 7 \times 6!}{6!(2 \times 1)} = \frac{8 \times 7}{2} = 28$

10. We have,

    $\frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!}$ $[\because n! = n(n-1)(n-2)\ldots 1]$

    $= 8 \times 7 \times 6 \times 5$ = 1680

    Again, $2! = 2 \times 1 = 2 \neq$ 1680

    $\therefore \frac{8!}{4!} \neq 2!$

11. According to the problem, 2-digits i.e. 6 and 7 are fixed. Thus, 10 - 2 = 8-digits can be

used in constructing the telephone numbers. There are 8-digits 0, 1, 2, 3, 4, 5, 8, 9. The first number can be selected in 8 ways. After the selection of the first digit, we have 8 - 1 = 7-digits in hand, second digit can be selected in 7 ways and the third digit can be selected in 6 ways.

According to the fundamental principle of multiplication (FPM), the number of ways of selecting a digit for remaining three places:

$= 8 \times 7 \times 6$ = 336 ways

12. We have, ${}^nP_r$ = P(n, r) $= \frac{n!}{(n-r)!}$

    i. ${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2!}{2!}$ [∵ n! = n(n - 1) (n - 2)!]
    $= 5 \times 4 \times 3$ = 60

    ii. P(15, 3) = ${}^{15}P_3 = \frac{15!}{(15-3)!} = \frac{15!}{12!}$
    $= \frac{15 \times 14 \times 13 \times 12!}{12!}$
    $= 15 \times 14 \times 13$
    = 2730

13. In choosing the families, there are the following cases:

    Case I Selecting 18 families from 52 families and 2 families from 87 - 52 = 35 families.

    Case II Selecting 19 families from 52 families and one from 35 families.

    Case III Selecting all the 20 families from 52 families.

    If P_1 , P_2 and P_3 are the respective selections in each case, then

    P_1 = ${}^{52}C_{18} \times {}^{35}C_2$

    P_2 = ${}^{52}C_{19} \times {}^{35}C_1$

    P_3 = ${}^{52}C_{20}$

    If P is the total number of choosing 20 families, then

    P = P_1 + P_2 + P_3

    = ${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$

14. In a word 'HONESTY', there are 7 letters and these letters can be arranged is ${}^7P_7$ ways.

    $= 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

    = 5040

I do not like jumbled words.

Honesty plays very important role in our life and people respect us for it.

15. We have,

p (15, r -1) = p (16, r - 2) = 3 : 4

$\Rightarrow \frac{P(15,r-1)}{P(16,r-2)} = \frac{3}{4}$

$\Rightarrow \frac{\frac{15!}{[15-(r-1)]!}}{\frac{16!}{[16-(r-2)]!}} = \frac{3}{4}$

$\Rightarrow \frac{\frac{15!}{[16-r]!}}{\frac{16!}{[18-r]!}} = \frac{3}{4}$

$\Rightarrow \frac{15!}{(16-r)!} \times \frac{(18-r)!}{16!} = \frac{3}{4}$

$\Rightarrow \frac{15! \times (18-r)(17-r)(16-r)!}{(16-r)! \times 16 \times 15!} = \frac{3}{4}$

$\Rightarrow \frac{(18-r)(17-r)}{16} = \frac{3}{4}$

$\Rightarrow 306 - 18r - 17r + r^2 = \frac{3}{4} \times 16$

$\Rightarrow r^2 - 35r + 306 = 12$

$\Rightarrow r^2 - 35r + 306 - 12 = 0$

$\Rightarrow r^2 - 35r + 294 = 0$

$\Rightarrow r^2 - 21r - 14r + 294 = 0$

$\Rightarrow r(r - 21) - 14(r - 21) = 0$

$\Rightarrow (r - 21)(r - 14) = 0$

$\Rightarrow r - 14 = 0$ [$\because r = 21 \neq 0$]

$\Rightarrow r = 14$

Hence, r = 14