CBSE Test Paper 02

CH-05 Complex & Quadratic

Section A

1. The inequality $|z - 6| < |z - 2|$ represents the region given by
   a. Re(z) > 4
   b. Re(z) < 2
   c. none of these
   d. Re(z) > 2
2. Find Argument of the complex number (0 + 0 i)
   a. $-\pi$
   b. π
   c. none of these
   d. 0
3. The least value of n for which $\left(\frac{1+i}{1-i}\right)^n$ is a positive integer is
   a. 8
   b. 1
   c. 2
   d. 4
4. If z is any complex number, then $\frac{z-\bar{z}}{2i}$ is
   a. either 0 or purely imaginary
   b. purely imaginary

c. purely real

d. none of these

5. The points z = x + iy which satisfy the equation $|z| = 1$ lie on

a. the line x = 1

b. the line y = 1

c. the line x + y = 1

d. the circle whose centre is origin and radius = 1

6. Fill in the blanks:

The roots of the equation $x^2 + 4 = 0$ are ________.

7. Fill in the blanks:

$5(\cos 270^{o} + i \sin 270^{o})$ is written in cartesian form as ________.

8. Evaluate $\frac{1}{i^7}$.

9. Express (5 + 4 i) + (5 - 4 i) in the form of a + ib.

10. Solve the inequalities: $2 \leqslant 3x - 4 \leqslant 5$

11. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers, then find arg $\left(\frac{z_1}{z_4}\right)$ + arg $\left(\frac{z_2}{z_3}\right)$.

12. If arg (z - 1) = arg (z + 3i), then find x - 1 : y.

13. Find the square root of $3 - 4\sqrt{7}i$

14. Find the real numbers x and y if (x - iy) (3+ 5i) is the conjugate of -6 - 24i.

15. Express the complex number $3 (\cos 300^{o} - i \sin 30^{o})$ in polar form.

Solution

1. (a) Re(z) > 4

   Explanation: Given

   Putting z=x+iy ,we get

   $|\text{x+iy} - 6| < |\text{x+iy} - 2|$

   $\Rightarrow |(\text{x} - 6)\text{+iy}| < |(\text{x} - 2)\text{+iy}|$

   $\Rightarrow \sqrt{(x-6)^2+y^2} < \sqrt{(x-2)^2+y^2}$ $\quad [\because |x+iy| = \sqrt{x^2+y^2}]$

   Squaring on both sides we get
   $(x-6)^2+y^2 < (x-2)^2+y^2$
   $\Rightarrow x^2+36-12x+y^2 < x^2+4-4x+y^2$
   $\Rightarrow 36-4 < 12x-4x$
   $\Rightarrow 32 < 8x$
   $\Rightarrow x > 4$

2. (c) none of these

   Explanation:

   Let $Z = 0+0i = r\left(cos\theta + isin\theta\right)$

   Then comparing the real and imaginary parts ,we get
   $r\cos\theta = 0$ and $r\sin\theta = 0$
   $\therefore r^2\left(\cos^2\theta+\sin^2\theta\right) = 0$
   $\Rightarrow r^2 = 0 \Rightarrow r = 0$

$\Rightarrow cos\theta = 0 \quad and \quad sin\theta = 0$

Which is not possible for any value of θ.

3. (d) 4

   Explanation:

   $\frac{1+i}{1-i} = \frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(1+i)^2}{1+1} = \frac{1-1+2i}{1+1} = \frac{2i}{2} =$

   $\therefore \left(\frac{1+i}{1-i}\right)^n = i^n$

   We have $i^4 = 1$ which is positive Hence the least of is 4

4. (c) purely real

   Explanation:

   Let $Z = x + iy$

   Then $\bar{Z} = x - iy$

   $\therefore Z - \bar{Z} = \quad (x + iy) - (x - iy) = 2iy$

   $Now \quad \frac{Z-\bar{Z}}{2i} = y$

   Hence $\frac{z-\bar{z}}{2i}$ is purely real

5. (d) the circle whose centre is origin and radius = 1

   Explanation:

   $z = x + iy$

   $Now \quad |z| = 1 \Rightarrow |x + iy| = 1$

   $\Rightarrow \sqrt{x^2 + y^2} = 1 \Rightarrow x^2 + y^2 = 1$

   which is the equation of a circle whose centre is origin and radius = 1

6. 2i and -2i

7. 0 - i5

8. Given, $\frac{1}{i^7} = \frac{1}{(i)^{4+3}} = \frac{1}{i^4 \cdot i^3} = \frac{1}{(1)i^2 \cdot i}$ [$\because i^4 = 1$, i^2=-1]

= $\frac{1}{(-1)i}$

multiplying numerator and denominator by i

$\Rightarrow \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2}$
= $\frac{i}{-(-1)} = i$

9. Given, $(5+4i)+(5-4i)$
= (5 + 5) + i(4 - 4) = 10 + 0i

10. We have $2 \leqslant 3x-4 \leqslant 5$
$\Rightarrow 2+4 \leqslant 3x \leqslant 5+4 \Rightarrow 6 \leqslant 3x \leqslant 9$
$\Rightarrow 2 \leqslant x \leqslant 3$

11. We have, $z_2 = \overline{z}_1$ and $z_4 = \overline{z}_3$

Therefore, $z_1 z_2 = |z_1|^2$ and $z_3 z_4 = |z_3|^2$

Now, arg $\left(\frac{z_1}{z_4}\right)$ + arg $\left(\frac{z_2}{z_3}\right)$ = arg $\left(\frac{z_1 z_2}{z_4 z_3}\right)$ [∵ Using formula, arg (z_1) + arg (z_2) = arg (z_1, z_2)]

= arg $\left(\frac{z_1 \overline{z}_1}{z_3 \overline{z}_3}\right)$ = arg $\left(\frac{|z_1|^2}{|z_3|^2}\right)$ = arg $\left(\left|\frac{z_1}{z_3}\right|^2\right)$ = 0 [∵ argument of positive real number is zero]

12. We have , arg (z - 1) = arg (z + 3i)
On putting z = x + iy, we get
arg (x + iy - 1) = arg (x + iy + 3i)
⇒ arg [(x - 1) + iy] = arg [x + i (y + 3)]
⇒ $\tan^{-1}\left(\frac{y}{x-1}\right)$ = $\tan^{-1}\left(\frac{y+3}{x}\right)$
$\Rightarrow \frac{y}{x-1} = \frac{y+3}{x}$ ⇒ xy = (x - 1) (y + 3)
⇒ xy = xy + 3x - y - 3
⇒ 0 = 3 (x - 1) - y
⇒ y = 3 (x - 1) ⇒ $\frac{x-1}{y} = \frac{1}{3}$
⇒ (x - 1) : y = 1 : 3

13. Let $x + yi = \sqrt{3-4\sqrt{7}i}$

Squaring both sides, we get

$x^2 - y^2 + 2xyi = 3 - 4\sqrt{7}i$

Equating the real and imaginary parts

$x^2 - y^2 = 3$ . . . (i)

$2xy = -4\sqrt{7} \Rightarrow xy = -2\sqrt{7}$

Now from the identity, we know

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= (3)^2 + 4(-2\sqrt{7})^2$

= 9 + 112

= 121

$\therefore x^2 + y^2 = 11$ . . . (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]

Solving (i) and (ii) we get

$x^2 = 7$ and $y^2 = 4$

$\therefore x = \pm\sqrt{7}, y = \pm 2$

Since the sign of xy is (-)

$\therefore$ if $x = \sqrt{7}$ y = -2

and if $x = \sqrt{7}$ y = 2

$\therefore \sqrt{3 - 4\sqrt{7}i} = \pm(\sqrt{7} - 2i)$

14. Here $\overline{-6 - 24i} = -6 + 24i$

Now (x - iy) (3 + 5i) = -6 + 24i

$\Rightarrow 3x + 5xi - 3yi - 5yi^2 = 6 + 24i$

$\Rightarrow (3x + 5y) + (5x - 3y)i = -6 + 24i$

Comparing both sides, we have

3x + 5y = -6. . . . (i)

and 5x - 3y = 24 .... (ii)

Multiplying (i) by 3 and (ii) by 5 and then adding

$$\begin{array}{l} 9x + 15y = -18 \\ \underline{25x - 15y = 120} \Rightarrow x = 3 \\ \quad 34x = 102 \end{array}$$

Putting x = 3 in (i)

3(3)+5y=-6

Thus y=-3

15. Let z = 3 (cos 300^o - i sin 30^o)

= 3 [cos (360^o - 60^o) - i sin 30^o]

= 3 [cos 60^o - i sin 30^o]

= 3 $\left[\frac{1}{2} - \frac{i}{2}\right]$

= $\frac{3}{2}$ (1 - i)

Let z = $\frac{3}{2}$ (1 - i) = r (cosθ + i sinθ)

On equating real and imaginary parts, we get

r cosθ = $\frac{3}{2}$ ...(i)

and r sinθ = - $\frac{3}{2}$ ...(ii)

On squaring and addng Eqs. (i) and (ii), we get

r^2 ($\cos^2\theta$ + $\sin^2\theta$) = $(\frac{3}{2})^2$ + $(-\frac{3}{2})^2$

$\Rightarrow r^2 = \frac{9}{4}$

$\therefore r = \frac{3}{\sqrt{2}}$

On putting the value of r in Eqs. (i) and (ii), we get

cosθ = $\frac{1}{\sqrt{2}}$ and sinθ = - $\frac{1}{\sqrt{2}}$

Since, cosθ is positive and sinθ is negative.

So, θ lies in IV quadrant.

$\therefore \theta = \frac{-\pi}{4}$

Polar form of z = $\frac{3}{\sqrt{2}}$ [cos $\left(-\frac{\pi}{4}\right)$ + i sin $\left(-\frac{\pi}{4}\right)$]

= $\frac{3}{\sqrt{2}}$ [cos $\frac{\pi}{4}$ - i sin $\frac{\pi}{4}$]

$\therefore$ 3 (cos 300^o - i sin 30^o) = $\frac{3}{\sqrt{2}}$ (cos $\frac{\pi}{4}$ - i sin $\frac{\pi}{4}$)