CBSE Test Paper 01

CH-05 Complex & Quadratic

1. The complex numbers z = x + iy ; x , y $\in$ R which satisfy the equation $\left|\frac{z-3i}{z+3}\right| = 1$ lies on
   a. the y axis
   b. the x axis
   c. the line x + y = 0
   d. the line parallel to y axis
2. If $\left(\sqrt{3}+i\right)^{10} = a + ib; a, b \in R,$ then a and b are respectively :
   a. 64 and - 64$\sqrt{3}$
   b. 512 and - 512$\sqrt{3}$
   c. 128 and 128$\sqrt{3}$
   d. none of these
3. z + $\overline{z} \neq 0$ if and only if
   a. z $\neq$ 0
   b. Re(z) $\neq$ 0
   c. Im (z) $\neq$0
   d. $|$ z $|$ $\neq$ 0
4. If $\alpha = \frac{z}{\overline{z}}$, then $|\alpha|$ is equal to :
   a. -1
   b. 0

c. 1

d. none of these

5. $1 + i^2 + i^4 + i^6 + i^8 + \ldots\ldots.$ up to 1001 terms is equal to

a. none of these

b. 0

c. 1

d. -1

6. Fill in the blanks:

The modulus and argument of z = 1 + i tanα is _______ and _______ respectively.

7. Fill in the blanks:

The value of $\frac{1}{i^7}$ is _______.

8. Solve $x^2 + 3 = 0$

9. Express the complex numbers (1 + i) - (- 1 + i6) in standard form

10. Find the difference of the complex numbers (6 + 5i), (3 +2i).

11. Express the complex numbers $\left(\frac{1}{5} + \frac{2}{5}i\right) - \left(4 + \frac{5}{2}i\right)$ in standard form

12. Solve: $ix^2 + 4x - 5i = 0$.

13. Find the square root of 1 - i.

14. If Re (z^2) = 0, |z| = 2, then prove that z = $\pm\sqrt{2} \pm i\sqrt{2}$.

15. Find all non-zero complex numbers of z satisfying $\overline{z} = iz^2$.

Solution

1. (c) the line x + y = 0

   Explanation: Let z=x+iy

   Now $\left|\frac{z-3i}{z+3}\right|=1$
   $\Rightarrow |z-3i|=|z+3|$
   $\Rightarrow |(x+iy)-3i|=|x+iy+3|$
   $\Rightarrow |x+i(y-3)|=|(x+3)+iy|$
   $\Rightarrow \sqrt{(y-3)^2+x^2}=\sqrt{(x+3)^2+(y)^2}$
   $\Rightarrow (y-3)^2+x^2=(x+3)^2+(y)^2$
   $\Rightarrow y^2-6y+9+x^2=x^2+6x+9+y^2$
   $\Rightarrow 6x+6y=0$
   $\Rightarrow x+y=0$

2. (b) 512 and - 512$\sqrt{3}$

   Explanation:

   First we will find the polar representation of the complex number $\sqrt{3}+i$
   Let $\sqrt{3}+i=r(\cos\theta+i\sin\theta) \Rightarrow r\cos\theta=\sqrt{3}and \quad r\sin\theta=1$
   $\therefore r^2\left(\cos^2\theta+\sin^2\theta\right)=3+1=4 \Rightarrow r^2=4 \Rightarrow r=2$

   Now $cos\theta=\frac{\sqrt{3}}{2}, \quad sin\theta=\frac{1}{2}$, both are positive .

   So Amplitude $=\theta=\frac{\Pi}{6}$

   Hence $\sqrt{3}+i=2\left(cos\frac{\Pi}{6}+isin\frac{\Pi}{6}\right)=2e^{\frac{i\Pi}{6}}$

   Now,
   $(\sqrt{3}+i)^{10}=\left(2e^{\frac{i\pi}{6}}\right)^{10}=2^{10}e^{\frac{i5\pi}{3}}=2^{10}\left(\cos\left(\frac{5\Pi}{3}\right)+i\sin\left(\frac{5\Pi}{3}\right)\right)$
   $=2^{10}\left(\cos\left(2\Pi-\frac{\Pi}{3}\right)+i\sin\left(2\Pi-\frac{\Pi}{3}\right)\right)=2^{10}\left(\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)\right)$

$$= 2^{10}\left(\cos\left(\frac{\Pi}{3}\right) - i\sin\left(\frac{\Pi}{3}\right)\right) = 2^{10}\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = 2^{10}\left(\frac{1-\sqrt{3}i}{2}\right) = 2^9(1-\sqrt{3}i)$$

Hence $\left(\sqrt{3}+i\right)^{10} = a + ib \Rightarrow 2^9\left(1-\sqrt{3}i\right) = a + ib$

$\Rightarrow a = 2^9 = 512 \quad and \quad b = -2^9\sqrt{3} = -512\sqrt{3}$

3. (b) Re(z) $\neq$ 0

   Explanation: Let Z=x+iy then we have

   So $Z + \bar{Z} = 2x$

   Now z + $\overline{z} \neq 0$
   $\Leftrightarrow 2x \neq 0$
   $\Leftrightarrow x \neq 0$
   $\Leftrightarrow Re(z) \neq 0$

4. (c) 1

   Explanation:

   Given $\alpha = \frac{z}{\overline{z}}$

   Then $|\alpha| = \left|\frac{z}{\overline{z}}\right| = \frac{|z|}{|\overline{z}|} = 1$ $\left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, |z| = |\overline{z}|\right]$

5. (c) 1

   Explanation:
   $1 + i^2 + i^4 + i^6 + i^8 + \ldots\ldots\ldots.$ upto 1001 terms
   $= \left(i^2\right)^0 + \left(i^2\right)^1 + \left(i^2\right)^2 + \left(i^2\right)^3 + \ldots\ldots.$ upto 1001 terms
   $= \left(i^2\right)^0 + \left(i^2\right)^1 + \left(i^2\right)^2 + \left(i^2\right)^3 + \ldots\ldots\ldots + \left(i^2\right)^{1000}$
   $= \left[\left(i^2\right)^0 + \left(i^2\right)^1\right] + \left[\left(i^2\right)^2 + \left(i^2\right)^3\right] + \ldots\ldots\ldots\ldots + \left[\left(i^2\right)^{998} + \left(i^2\right)^{999}\right] + \left[\left(i^2\right)\right.$
   $= [1-1] + [1-1] + \ldots\ldots\ldots + [1-1] + 1$
   =1

6. secα, α

7. i

8. Here $x^2 + 3 = 0 \Rightarrow x^2 = -3 \Rightarrow x = \pm\sqrt{-3} = \pm\sqrt{3}i$

9. (1 + i) - (-1 + i6)

   1 + i + 1 - 6i = 2 - 5i

10. (6 + 5i) - (3 + 2i) = (6 + 5i) + (- 3 - 2i)

    = (6 - 3) + (5 - 2) i = 3 + 3i

11. $\left(\frac{1}{5} + \frac{2}{5}i\right) - \left(4 + \frac{5}{2}i\right)$
    $= \frac{1}{5} + \frac{2}{5}i - 4 - \frac{5}{2}i$
    $= \left(\frac{1}{5} - 4\right) + \left(\frac{2}{5} - \frac{5}{2}\right)i$
    $= \frac{-19}{5} - \frac{21}{10}i$

12. We have, $ix^2 + 4x - 5i = 0$ ...(i)

    On comparing Eq. (i) with $ax^2 + bx + c = 0$, we get

    a = i, b = 4 and c = - 5i

    $\because x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

    $\therefore x = \frac{-4\pm\sqrt{(4)^2-4\times i(-5i)}}{2\times i}$

    $= \frac{-4\pm\sqrt{16+20i^2}}{2i}$

    $= \frac{-4\pm\sqrt{16-20}}{2i}$ [$\because i^2 = -1$]

    $= \frac{-4\pm\sqrt{-4}}{2i} = \frac{-4\pm 2i}{2i}$

    $= \frac{4i^2\pm 2i}{2i}$ = $2i \pm 1$ [$\because -1 = i^2$]

    $\therefore$ x = 2i + 1 and x = 2i - 1

    Hence, the roots of the given equation are 2i + 1 and 2i - 1.

13. Let $x + yi = \sqrt{1 - i}$

    Squaring both sides, we get

    $x^2 - y^2 + 2xyi = 1 - i$

    Equating the real and imaginary parts

    $x^2 - y^2 = 1$ and $2xy = -1$... . (i)

    $\therefore\ xy = \frac{-1}{2}$

    Using the identity

$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= (1)^2 + 44\left(-\frac{1}{2}\right)^2$

= 1 + 1

= 2

$\therefore\ x^2 + y^2 = \sqrt{2}\ \ldots.$ (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]

Solving (i) and (ii) we get

$x^2 = \frac{\sqrt{2}+1}{2}$ and $y = \frac{\sqrt{2}-1}{2}$

$\therefore x = \pm\sqrt{\frac{\sqrt{2}+1}{2}}$ and $y = \pm\sqrt{\frac{\sqrt{2}-1}{2}}$

Since the sign of xy is negative.

$\therefore$ if $x = \sqrt{\frac{\sqrt{2}+1}{2}}$ then $y = -\sqrt{\frac{\sqrt{2}-1}{2}}$

and if $x = -\sqrt{\frac{\sqrt{2}+1}{2}}$ then $y = \sqrt{\frac{\sqrt{2}-1}{2}}$

$\therefore \sqrt{1-i} = \pm\left(\sqrt{\frac{\sqrt{2}+1}{2}} - \sqrt{\frac{\sqrt{2}-1}{2}}i\right)$

14. Let z = x + iy

$\Rightarrow z^2 = (x + iy)^2$ [squaring both sides]

$\Rightarrow z^2 = x^2 + i^2 y^2 + 2\, ixy$

$\Rightarrow z^2 = (x^2 - y^2) + i\,(2xy)$

$\therefore$ Re $(z^2) = 0$

$\Rightarrow x^2 - y^2 = 0 \Rightarrow x = \pm\ y$

$\Rightarrow x = y$ ...(i)

and $x = -\ y$ ...(ii)

Again, $|z| = 2$ [given]

$\Rightarrow\ |z|^2 = 4$

$\Rightarrow x^2 + y^2 = 4$ ...(iii)

From Eqs. (i) and (iii), we get

$y^2 + y^2 = 4 \Rightarrow\ 2y^2 = 4$

$\Rightarrow y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

Therefore, from Eq. (i), we get

$x = \ \pm\sqrt{2}$

$\therefore z = \pm\sqrt{2} \pm i\sqrt{2}$

On putting the value of x from Eq. (ii) in Eq. (iii), we get

$(-y)^2 + y^2 = 4 \Rightarrow 2y^2 = 4$

$\Rightarrow y^2 = 2 \Rightarrow y = \pm\sqrt{2}$

From Eq. (ii), $x = \pm\sqrt{2}$

$\therefore z = x + iy \Rightarrow z = \pm\sqrt{2} \pm i\sqrt{2}$

Hence proved.

15. Let $z = x + iy$

Given: $\overline{z} = iz^2$

$\Rightarrow x - iy = i(x^2 - y^2 + 2i\,xy)$

$\Rightarrow x - iy = i(x^2 - y^2) - 2xy$

$\Rightarrow (x + 2\,xy) - i\,(x^2 - y^2 + y) = 0$

$\Rightarrow x + 2xy = 0$ ...(i) and $x^2 - y^2 + y = 0$ ...(ii)

Now,

$x + 2\,xy = 0 \Rightarrow x\,(1 + 2y) = 0 \Rightarrow x = 0$ or $1 + 2y = 0 \Rightarrow x = 0$ or $y = -\frac{1}{2}$

CASE I: When $x = 0$

Putting $x = 0$ in (ii), we have

$\Rightarrow -y^2 + y = 0 \Rightarrow y(y - 1) = 0 \Rightarrow y = 0, y = 1$

Thus, we have the following pairs of values of x and y :

$x = 0, y = 0; x = 0, y = 1$

$\therefore z = 0 + i\,0 = 0$ and $z = 0 + 1i = i$

CASE II: When $y = -\frac{1}{2}$

Putting $y = \frac{-1}{2}$ in (ii), we get

$x^2 - y^2 + y = 0 \Rightarrow x^2 - \frac{1}{4} - \frac{1}{2} = 0 \Rightarrow x^2 - \frac{3}{4} = 0 \Rightarrow x = \pm \frac{\sqrt{3}}{2}$

Thus, we have the following pairs of values of x and y:

$x = \frac{\sqrt{3}}{2}, y = \frac{-1}{2}$ and $x = \frac{-\sqrt{3}}{2}, y = \frac{-1}{2}$

$\therefore z = \frac{\sqrt{3}}{2} - \frac{1}{2}i$ and $z = \frac{-\sqrt{3}}{2} - \frac{1}{2}i$

Hence, all non-zero complex numbers of z are $i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i$