CBSE Test Paper 02

CH-03 Trigonometric Functions

1. If in a triangle ABC, $a\cos^2\left(\frac{C}{2}\right)+c\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$ then its sides a, b, c are in
   a. G.P.
   b. none of these
   c. A. P.
   d. H.P.
2. Angles of triangle are in A.P. If the number of degrees in the smallest to the number of radians in the largest is as $60:\pi$, then the smallest angle is
   a. none of these.
   b. 40°
   c. 20°
   d. 30°
3. If the sides of a triangle are 13, 7, 8 the greatest angle of the triangle is
   a. $\frac{2\pi}{3}$
   b. $\frac{\pi}{2}$
   c. $\frac{3\pi}{2}$
   d. $\frac{\pi}{3}$
4. The largest value of $\sin\theta\cos\theta$ is
   a. $\frac{1}{\sqrt{2}}$
   b. $\frac{\sqrt{3}}{2}$
   c. $\frac{1}{2}$
   d. 1
5. ABC is an equilateral triangle of each side a (>0). The inradius of the triangle is
   a. $\frac{a}{3}$
   b. $\frac{\sqrt{3}\,a}{3}$
   c. $\frac{a}{2}$
   d. $\frac{a}{2\sqrt{3}}$
6. Fill in the blanks:

   The solution of a trigonometric equation for which the value of unknown angle say x lies between 0 and 2π is called its ________ solution.

7. Fill in the blanks:

   The minute hand rotates through an angle of 6^0 in ________ minute.

8. Express as a product: sin 6x - sin 2x

9. Find the values of the trigonometric function: $\cot\left(\frac{-15\pi}{4}\right)$

10. Find the value of 2sin 15°. cos75°

11. Find the general solutions of the following equations:

    i. cos 3x = 0
    ii. $\cos\frac{3x}{2} = 0$
    iii. $\cos^2 3x = 0$

12. Find the principal solution of the equation tan x = $\frac{-1}{\sqrt{3}}$

13. Prove $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$

14. Prove that : sin x + sin 3x + sin 5x + sin 7x = 4 cos x cos 2x sin 4x

15. Prove that: sin 20^0 sin 40^0 sin 80^0 = $\frac{\sqrt{3}}{8}$

Solution

1. (c) A. P.

   Explanation:

   $a\cos^2\left(\frac{C}{2}\right)+\cos^2\left(\frac{A}{2}\right)=\frac{3b}{2}$

   $\Rightarrow a\left(\frac{1+\cos C}{2}\right)+c\left(\frac{1+\cos A}{2}\right)=\frac{3b}{2}$

   $\Rightarrow \frac{1}{2}(a\cos C+\text{c}\cos A+a+c)=\frac{3b}{2}$

   $\Rightarrow \frac{1}{2}(b+a+c)=\frac{3b}{2}$

   $[\because \quad a.\cos C+c.cosA=b \quad \text{using} \quad \text{Projection} \quad \text{rule} \quad]$

   $\Rightarrow \bar{a}+c=2b$

   Which means they are in A.P

2. (d) 30°

   Explanation:

   Let the measures of the angles of the triangle be a-d, a, a+d degrees respectively

   Then we have $a-d+a+a+d=180^\circ$

   $\Rightarrow 3a=180^\circ$

   $\Rightarrow a=\frac{180^\circ}{3}=60^\circ$

   But it is clear that $a-d=60^\circ-d$ is the least and $a+d=60^\circ+d$ is the greatest angle.

   Given greatest angle = π radian $=180^\circ$

   $\frac{\text{least angle}}{\text{greatest angle}}=\frac{60^\circ}{\pi\text{radian}}$

   $\Rightarrow \frac{60^\circ-d}{60^\circ+d}=\frac{60^\circ}{180^\circ}=\frac{1}{3}$

   $\Rightarrow 180^\circ-3d=60^\circ+d$

   $\Rightarrow 4d=120^\circ \Rightarrow d=30^\circ$

3. (a) $\frac{2\pi}{3}$

Explanation: Let the sides of the triangle be a=13,b=7, and c=8.

Since a is the longest side the greatest angle is A.

Using Cosine rule, we have

$$CosA = \frac{b^2+c^2-a^2}{2bc} = \frac{49+64-169}{112} = \frac{-1}{2}$$
But we have cos $\cos\left(\frac{2\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = \frac{-1}{2} \therefore \quad A = \frac{2\pi}{3}$

4. (c) $\frac{1}{2}$

   Explanation: $\sin\theta\cos\theta = \frac{1}{2} \cdot 2\sin\theta\cos\theta = \frac{1}{2} \cdot \sin 2\theta$

   But the maximum value of $\sin 2\theta$ is 1.

   So the maximum value of $\sin\theta\cos\theta = \frac{1}{2}$

5. (d) $\frac{a}{2\sqrt{3}}$

   Explanation: If r is the inradius of the circle then we have, $r = \frac{\Delta}{s}$
   Given A B C is an equilateral triangle, so $\Delta = \sqrt{s(s-a)(s-b)(s-c)} \Rightarrow$
   $\Delta = \sqrt{s(s-a)(s-a)(s-a)}$
   $\because r = \frac{\Delta}{s} = \frac{s-a}{s}\sqrt{s(s-a)}$
   Also, $s = \frac{3a}{2}\ [\because \quad 2s = a+b+c \quad]$
   $\Rightarrow r = \frac{\frac{3a}{2}-a}{\frac{3a}{2}}\sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)}$
   $\Rightarrow r = \frac{a}{2} \cdot \frac{2}{3a}\sqrt{\frac{3a^2}{4}} = \frac{a}{2\sqrt{3}}$

6. principal

7. one

8. sin 6x - sin 2x
   $= 2\sin\left(\frac{6x-2x}{2}\right)\cos\left(\frac{6x+2x}{2}\right)\ \left[\because \sin C - \sin D = 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}\right]$
   = 2 sin 2x cos 4x

9. $\cot\left(\frac{-15\pi}{4}\right) = \cot\left(\frac{-15\times180^\circ}{4}\right) = \cot(-675^\circ)$
   $\cot(-675^\circ) = \cot(-2\times360^\circ + 45^\circ)$ = cot 45° = 1

10. Given,

$2sin15^o cos75^o = sin(15^o + 75^o) + sin(15^o - 75^o)$

[$\because$ $2\sin x \cdot \cos y$ = sin (x + y) + sin (x - y)]

$= sin90^o + sin(-60^o)$

= sin 90^0 - sin 60^0 [$\because$ sin (- θ) = - sin θ]

= 1 - $\frac{\sqrt{3}}{2}$ = $\frac{2-\sqrt{3}}{2}$

11. As we know that the general solution of the equation $cos\ x = 0$ is $x = (2n+1)\frac{\pi}{2}, n \in$Z.

$\therefore$

i. $cos3x = 0 \Rightarrow 3x = (2n+1)\frac{\pi}{2}, n \in Z \Rightarrow x = (2n+1)\frac{\pi}{6}, n \in Z$

ii. $\cos\frac{3x}{2} = 0 \Rightarrow \frac{3x}{2} = (2n+1)\frac{\pi}{2}, n \in Z \Rightarrow x = (2n+1)\frac{\pi}{3}, n \in Z$

iii. $\cos^2 3x = 0 \Rightarrow \cos 3x = 0 \Rightarrow 3x = (2n+1)\frac{\pi}{2},$
$n \in Z \Rightarrow x = (2n+1)\frac{\pi}{6}, n \in Z.$

12. Given, $tan\ x = \frac{-1}{\sqrt{3}}$

Here, value of $tan\ x$ is negative. So, x lies in II and IV quadrants.

As we know, $tan\ \frac{\pi}{6}$ = $\frac{-1}{\sqrt{3}}$

$\therefore tan\left(\pi - \frac{\pi}{6}\right)$ = - $\frac{1}{\sqrt{3}}$ $\Rightarrow \tan\frac{5\pi}{6} = \frac{-1}{\sqrt{3}}$

$\therefore x = \frac{5\pi}{6}$, which lies in I quadrant.

$tan\left(2\pi - \frac{\pi}{6}\right)$ = - tan $\frac{\pi}{6}$

$\Rightarrow \tan\frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$

$\therefore x = \frac{11\pi}{6}$, which lies in IV quadrant.

$\therefore$ principal solutions are $x = \frac{5\pi}{6}$ and $x = \frac{11\pi}{6}$.

13. We have,

L.H.S = $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{-(\sin 3x - \sin x)}{-(\cos^2 x - \sin^2 x)} = \frac{(\sin 3x - \sin x)}{(\cos^2 x - \sin^2 x)}$

$= \frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{\cos 2x}$

$= \frac{2\cos 2x \sin x}{\cos 2x} = 2\sin x$

14. We have L.H.S. = sin x + sin 3x + sin 5x + sin 7x

= (sin 7x + sin x) + (sin 5x + sin 3x)

$$= \left[2\sin\left(\frac{7x+x}{2}\right)\cos\left(\frac{7x-x}{2}\right)\right]+\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

= 2sin 4xcos3x + 2sin 4xcosx

$\left[\because \sin\ C+\sin D = 2\sin\frac{C+D}{2}\cdot\cos\frac{C-D}{2}\right]$

= 2 sin 4x [cos 3x + cos x]

$$= 2\sin 4x\left[2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)\right]$$

$\left[\because \cos C+\cos D = 2\cos\frac{C+D}{2}\cdot\cos\frac{C-D}{2}\right]$

= 2 sin 4x [2 cos 2x cos x]

= 4cos x cos 2x sin 4x = R.H.S.

15. Given, LHS $= sin20^o sin40^o sin80^o$

= $\frac{1}{2}$ [2 sin $20^\circ \cdot \sin 40^\circ$] sin 80^0 [multiplying and dividing by 2]

= $\frac{1}{2}\left[cos(20^o-40^o)-cos(20^o+40^o)\right]$ $\cdot \sin 80^\circ$ [$\because 2\sin x\cdot\sin y$ = cos (x - y) - cos (x + y)]

= $\frac{1}{2}$ $[cos(-20^o)-cos60^o]sin80^o$

= $\frac{1}{2}$ [cos 20^0 - $\frac{1}{2}$]$\cdot$ sin 80° [$\because$ cos (- θ) = cos θ and cos 60^0 = $\frac{1}{2}$]

= $\frac{1}{2}\times\frac{1}{2}\left[2\left(\cos 20^\circ-\frac{1}{2}\right)\cdot\sin 80^\circ\right]$ [again multiplying and dividing by 2]

= $\frac{1}{4}\left[2\cos 20^\circ\cdot\sin 80^\circ-\sin 80^\circ\right]$

= $\frac{1}{4}\left[\sin(20^\circ+80^\circ)-\sin(20^\circ-80^\circ)-\sin 80^\circ\right]$ [$\because 2\cos x\cdot\sin y$ $= sin(x+y)-sin(x-y)$]

= $\frac{1}{4}$ $[sin100^o-sin(-60^o)-sin80^o]$

= $\frac{1}{4}$ [sin 100^0 + sin 60^0 - sin 80^0] [$\because$ sin (- θ) = - sin θ]

= $\frac{1}{4}$ [sin (180^0 - 80^0) + sin 60^0 - sin 80^0] [$\because$ sin 100^0 = sin (180^0 - 80^0)]

= $\frac{1}{4}$ [sin 80^0 + sin 60^0 - sin 80^0] [$\because$ sin ($\pi-\theta$) = sin θ]

= $\frac{1}{4}$ $\times$ sin 60^0 = $\frac{1}{4}$ $\times$ $\frac{\sqrt{3}}{2}$ [$\because$ sin 60^0 = $\frac{\sqrt{3}}{2}$]

= $\frac{\sqrt{3}}{8}$ = RHS

Hence proved.