CBSE Test Paper 01

CH-03 Trigonometric Functions

1. In a triangle ABC, if A = 75^o and B = 45^o then b + 2 c is equal to
   a. a
   b. a + b + c
   c. 2 a
   d. $\frac{1}{2}(a+b+c)$
2. cot θ = sin 2 $\theta(\theta \neq$n π , n integer) if θ equals
   a. 90° only
   b. $45^{\circ} and\ 60^{\circ}$
   c. 45° only
   d. $45^{\circ} and\ 90^{\circ}$
3. If the angles of a triangle ABC are in A.P., then
   a. none of these
   b. $c^2 = a^2 + b^2$
   c. $a^2 + c^2 - ac = b^2$
   d. $c^2 = a^2 + b^2 + ab$
4. In a triangle ABC, the line joining the circumcentre and the incentre is parallel to BC, then cos B + cos C =
   a. $\frac{3}{2}$
   b. 1
   c. $\frac{1}{2}$
   d. $\frac{3}{4}$
5. In a triangle ABC, AD is the median A to BC, then its length is equal to
   a. $\frac{b+c}{2}$
   b. $\sqrt{b^2 + c^2 - \frac{a^2}{2}}$
   c. $\sqrt{\frac{b^2+c^2-a^2}{2}}$
   d. $\frac{1}{2}\sqrt{2(b^2+c^2) - a^2}$
6. Fill in the blanks:

   If sinθ + cosecθ = 2, then sin$^2\theta$ + cosec$^2\theta$ = ________.

7. Fill in the blanks:

   The general solution for sec x = sec $(\pi + x)$ is ________.

8. Evaluate: cos (- 870^{o})

9. Express as a product: cos 4x + cos 8x

10. Find the degree corresponding to the radian measure-2^{c}

11. Prove $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

12. Solve: tan x + tan 2x + tan 3x = 0

13. If $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta}$ = $a^2 - b^2$ and, $\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta}$ = 0, prove that $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}}$ = $(a^2 - b^2)^{\frac{2}{3}}$.

14. A horse is tied to a post by a rope. If the horse moves along a circular path always keeping the rope tight and describes 66 m when it has traced out 45° at the centre, find the length of the rope.

15. Solve: 4 sinx cosx + 2 sinx + 2 cosx + 1 = 0.

Solution

1. (c) 2 a

   Explanation:

   Given $A = 75^\circ$ and $B = 45^\circ$
   then $A + B + C = 180^\circ$
   $\Rightarrow 75 + 45 + C = 180$
   $\Rightarrow$**C=180-120=60**
   From sin e formula
   $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k(\text{lc}\, t)$
   $\Rightarrow \frac{a}{\sin 75} = \frac{b}{\sin 45} = \frac{c}{\sin 60} = k$
   Here
   $\frac{a}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = k, \frac{b}{\frac{1}{\sqrt{2}}} = k, \frac{c}{\frac{\sqrt{3}}{2}} = k$**Now,**
   $\Rightarrow a = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) k, b = \frac{k}{\sqrt{2}}, c = \frac{\sqrt{3}k}{2}$
   $b + 2c = \frac{k}{\sqrt{2}} + 2 \cdot \frac{\sqrt{3}k}{2} = \frac{2k+2\sqrt{3}k}{2\sqrt{2}} = 2\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) k = 2a$

2. (d) $45^\circ and\ 90^\circ$

   Explanation:
   $\sin 2\theta = \cot \theta$
   $\Rightarrow 2 \sin \theta \cos \theta = \frac{\sin \theta}{\cos \theta}$
   $\Rightarrow \cos \theta \left(2 \sin \theta - \frac{1}{\sin \theta}\right) = 0$
   $\Rightarrow -\cos \theta \left(1 - 2 \sin^2 \theta\right) = 0$
   $\Rightarrow -\cos \theta \cdot \cos 2\theta = 0$
   $\Rightarrow \cos \theta = \cos \frac{\pi}{2} \quad \text{or} \quad \cos 2\theta = \cos \frac{\pi}{2}$
   $\Rightarrow \theta = \frac{\pi}{2}, \frac{\pi}{4} \quad [\because \quad \theta \neq n\pi \quad , n \in Z]$

3. (c) $a^2 + c^2 - ac = b^2$

Explanation:

Given angles of a triangle ABC are in A.P$\Rightarrow \frac{A+C}{2} = B \Rightarrow A + C = 2B....(i)$

But we have in a triangle $A + B + C = 180^{o} \Rightarrow 3B = 180^{o} \Rightarrow B = 60^{o}$ [using(i)]
Using Cosine Rule we have

$b^2 = a^2 + c^2 - 2ac.cosB$
$\Rightarrow b^2 = a^2 + c^2 - 2ac.cos60^\circ$
$\Rightarrow b^2 = a^2 + c^2 - ac \quad [\because \quad cos60^\circ = \frac{1}{2}]$

4. (b) 1

   Explanation:

   Using the given condition we have r = R cos A which gives r/R = cos A.hence Cos A + Cos B + cos C -1 = cos A which gives cos B + cos C = 1.

5. (d) $\frac{1}{2}\sqrt{2(b^2 + c^2) - a^2}$

   Explanation:

   Using the given information let the length of the median be d units, and let BD = DB = m units. Using Apollonius theorem we have the following results.
   From triangle ADC , if the angle ADC is θ, then $b^2 = m^2 + d^2 - 2mdcos\,\theta$. The angle ADB is the supplement of angle ADC and it will be - cos θ.
   Hence from triangle ADB we have $b^2 = m^2 + d^2 + 2mdcos\,\theta$. When we add both the results we have $b^2 + c^2 = 2d^2 + 2m^2 = 2d^2 + 2\left(\frac{a}{2}\right)^2 = 2d^2 + \left(\frac{a^2}{2}\right)$.
   On simplifying we get the result.

6. 2

7. x = $\frac{\pi}{2}(2n - 1)$

8. $cos(-870^o) = cos870^o\ [\because cos(-\theta) = cos\ \theta]$

   cos $\left(\frac{\pi}{2} \times 10 - 30\right)$ = $\pm$ cos 30^o [$\because$ n is even]
   Now, $\alpha = 870^o = \frac{\pi}{2} \times 10 - 30^o$
   It lies in II quadrant in which cos θ is negative.

So, cos (- 870^0) = - cos 30^0 = - $\frac{\sqrt{3}}{2}$

9. cos 4x + cos 8x

$= 2\cos\left(\frac{8x+4x}{2}\right)\cos\left(\frac{8x-4x}{2}\right)\ \left[\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right]$

= 2 cos 6x cos 2x

10. $(-2)^c = \left(\frac{180}{\pi} \times -2\right)^\circ = \left(\frac{180}{22} \times 7 \times (-2)\right)^\circ = \left(-114\frac{6}{11}\right)^\circ$

$= \left(-114^\circ\left(\frac{6}{11} \times 60\right)'\right)$

$= -\left[114^\circ\left(32\frac{8}{11}\right)'\right] = -\left[-114^\circ 32'\left(\frac{8}{11} \times 60\right)''\right]$

$= -[114^\circ 32' 44'']$

11. We have L.H.S. $= \sin^2 6x - \sin^2 4x$

$= \sin(6x+4x)\cdot\sin(6x-4x)$

$[\because \sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)]$

$= \sin 10x \cdot \sin 2x = R.H.S.$

12. tan x + tan 2x + tan 3x = 0

tan x + tan 2x + $\frac{(\tan x + \tan 2x)}{1 - \tan x \cdot \tan 2x} = 0$ [using tan 3x=$\frac{(\tan x + \tan 2x)}{1 - \tan x \cdot \tan 2x}$]

[tan x + tan 2x] $\left[1 + \frac{1}{1 + \tan x \tan 2x}\right] = 0$

[tan x + tan 2x][2 - tan x tan 2x] = 0

tan x = -tan 2x or tan x tan 2x = 2

$\mathrm{x} = \mathrm{n}\pi - 2\mathrm{x}$ or $\tan\mathrm{x} \cdot \frac{2\tan\mathrm{x}}{1+\tan^2\mathrm{x}} = 2$

$3\mathrm{x} = \mathrm{n}\pi$ or $2\tan^2\mathrm{x} = 2 - 2\tan^2\mathrm{x}$

$3\mathrm{x} = \mathrm{n}\pi$ or $4\tan^2\mathrm{x} = 2$

$\mathrm{x} = \frac{\mathrm{n}\pi}{3}$ or $\tan{}^2\mathrm{x} = \frac{1}{2}$

$\mathrm{x} = \frac{\mathrm{n}\pi}{3}$ or $\mathrm{x} = \mathrm{m}\pi + \tan^{-1}\left(\frac{1}{\sqrt{2}}\right), \quad n, m \in Z$

13. Given,

$\frac{ax\sin\theta}{\cos^2\theta} - \frac{by\cos\theta}{\sin^2\theta} = 0$

$\Rightarrow ax\ sin^3\ \theta$ - $by\ cos^3\ \theta = 0$

$\Rightarrow \frac{\sin^3\theta}{by} = \frac{\cos^3\theta}{ax}$

$\Rightarrow \left(\frac{\sin^3\theta}{by}\right)^{2/3} = \left(\frac{\cos^3\theta}{ax}\right)^{2/3}$

$\Rightarrow \frac{\sin^2\theta}{(by)^{2/3}} = \frac{\cos^2\theta}{(ax)^{2/3}}$

$\Rightarrow \frac{\sin^2\theta}{(by)^{2/3}} = \frac{\cos^2\theta}{(ax)^{2/3}} = \frac{\sin^2\theta+\cos^2\theta}{(by)^{2/3}+(ax)^{2/3}}$ [Using ratio and proportions]

$\Rightarrow \frac{\sin^2\theta}{(by)^{2/3}} = \frac{\cos^2\theta}{(ax)^{2/3}} = \frac{\sin^2\theta+\cos^2\theta}{(by)^{2/3}+(ax)^{2/3}} = \frac{1}{(by)^{2/3}+(ax)^{2/3}}$

$\Rightarrow sin^2\ \theta = \frac{(by)^{2/3}}{(ax)^{2/3}+(by)^{2/3}}$ and, $cos^2\theta = \frac{(ax)^{2/3}}{(ax)^{2/3}+(by)^{2/3}}$

$\Rightarrow sin\ \theta = \frac{(by)^{1/3}}{\sqrt{(ax)^{2/3}+(by)^{2/3}}}$ and, $cos\ \theta = \frac{(ax)^{1/3}}{\sqrt{(ax)^{2/3}+(by)^{2/3}}}$

Substituting the values in $\frac{ax}{\cos\theta} + \frac{by}{\sin\theta} = a^2 - b^2$,

$\Rightarrow (ax)^{2/3}\sqrt{(ax)^{2/3}+(by)^{2/3}} + (by)^{2/3}\sqrt{(ax)^{2/3}+(by)^{2/3}} = a^2 - b^2$

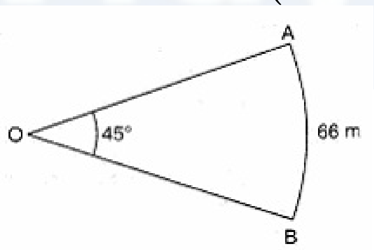
$\Rightarrow \left\{\sqrt{(ax)^{2/3}+(by)^{2/3}}\right\}\left\{(ax)^{2/3}+(by)^{2/3}\right\} = a^2 - b^2$

$\Rightarrow \left\{(ax)^{2/3}+(by)^{2/3}\right\}^{3/2} = a^2 - b^2 \Rightarrow (ax)^{2/3}+(by)^{2/3} = \left(a^2-b^2\right)^{2/3}$

14. Here PA = PB = r

arc AB = 66 m and $\theta = 45^\circ$

Now $\theta = 45^\circ = \left(45 \times \frac{\pi}{180}\right)^C = \frac{\pi^C}{4}$



We know that

$\theta = \frac{1}{r}$

$\therefore \frac{\pi}{4} = \frac{66}{r} \Rightarrow r = \frac{66\times4}{22} \times 7 = 84m$

15. $4\sin x\cos x + 2\sin x + 2\cos x + 1 = 0$

$\Rightarrow 2sin\ x(2cos\ x+1) + 1(2cos\ x+1) = 0$

$\Rightarrow (2sin\ x+1)(2cos\ x+1) = 0$

$\Rightarrow 2sin\ x + 1 = 0$

or $2cos\ x + 1 = 0$

$\Rightarrow sinx = -\frac{1}{2}$

or $cos x = -\frac{1}{2}$

Now, if sin x = - $\frac{1}{2}$

$\Rightarrow$ sin x = sin $\left(-\frac{\pi}{6}\right)$

$\therefore$ The general solution of this equation is

x = $n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$ = $n\pi + (-1)^{n+1}\left(\frac{\pi}{6}\right)$

$\Rightarrow$ x = $\pi\left[n + \frac{(-1)^{n+1}}{6}\right]$ ...(i)

and if cos x = $\frac{-1}{2}$

$\Rightarrow$ cos x = cos $\left(\pi - \frac{\pi}{3}\right)$ = cos $\frac{2\pi}{3}$

The general solution of this equation is

x = $2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow$ x = $2\pi\left(n \pm \frac{1}{3}\right)$ ... (ii)

From Eqs. (i) and (ii), we have x = $\pi\left[n + \frac{(-1)^{n+1}}{6}\right]$ or $2\pi\left(n \pm \frac{1}{3}\right)$ where n $\in$ Z

These are the required solutions.