CBSE Test Paper 02

CH-02 Relations and Functions

Section A

1. If f(x) = $\frac{x}{x-1} = \frac{1}{y}$, then f (y) =

   a. 1 + x

   b. 1 – x

   c. x – 1

   d. x

2. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 + y^2 = 144\}$, then $A \cap B$ contains

   a. one point

   b. two points

   c. four points

   d. one point

3. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by a car or bus is

   a. 60 percent

   b. 80 percent

   c. 70 percent

   d. 40 percent

4. The domain of the real-valued function $f(x) = \frac{(x-3)(x-1)}{\sqrt{x^2-4}}$ is

a. $(-\infty, -1) \cup (1, \infty)$.

b. $(-\infty, -2) \cup (1, \infty)$

c. $(-\infty, -2) \cup (2, \infty)$

d. (1, 2)

5. If f (x) = tan x, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and g (x) = $\sqrt{3 - x^2}$, then domain of the function gof is

   a. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

   b. $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

   c. $\left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$

   d. none of these

6. Fill in the blanks:

   The set of first elements of all the ordered pairs present in R is called the _____ of relation.

7. Fill in the blanks:

   If $f(x) = x^2 + 2x + 3$, then f(1) is _____.

8. Write the relation R= $\{(x, x^3): x \text{ is a prime number less than } 10\}$ in roster form.

9. If X = {0, ± 2, 4} and Y = {0, 4, 5, 16}, then represent the rule f : X $\rightarrow$ Y given by $f(x) = x^2$ by an arrow diagram.

10. A function f: R $\rightarrow$ R is defined by f (x) = x^2, $x \in R$. Determine range of f.

11. If A $\subseteq$ B, show that $A \times A \subseteq (A \times B) \cap (B \times A)$.

12. Find the domain of the function f defined by $f(x) = \sqrt{4 - x} + \frac{1}{\sqrt{x^2 - 1}}$.

13. If h denotes the number of honest people and p denotes the number of punctual

people and a relation between honest people and punctual people is given as h = p +16. If P denotes the number of people who progress in life and a relation between number of people who progress and honest people is given as

P = $\left(\frac{h}{8}\right)$ + 5

Find the relation between number of people who progress in life and punctual people. How does the punctuality is important in the progress of life?

14. Find the domain and range of the function $f(x) = \frac{x^2-9}{x-3}$

15. i. Let R be the relation on the set Z of all integers defined by R = {(x, y): x - y is divisible by n}. Prove that
    a. (x, y) $\in$ R
       $\Rightarrow$ (y, x) $\in$ R for all x, y $\in$ Z.
    b. (x, y) $\in$ R and (y, z) $\in$ R
       $\Rightarrow$ (x, z) $\in$ R for all x, y, z $\in$ Z.

    ii. Find the domain and range of the function $f(x) = \frac{x^2-9}{x-3}$.

    iii. Find the domain of the function $f(x) = \frac{x^2+3x+5}{x^2+x-6}$.

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Solution

Section A

1. (b) 1 – x

   Explanation:

   We have, $f(x) = \frac{x}{x-1} = \frac{1}{y}$

   $\therefore y = \frac{x-1}{x}$

   Now, $f(x) = \frac{x}{x-1}$

   $\Rightarrow f(y) = \frac{y}{y-1} = \frac{\frac{x-1}{x}}{\frac{x-1}{x}-1} = \frac{x-1}{x-1-x} = \frac{x-1}{-1} = 1 - x$

2. (c) four points

   Explanation:

   We will solve equations in A and B simultaneously and find values of x and y. The no. of possible ordered pairs from these values will be elements in $A \cap B$.

   Now, From B,$x^2 + 9y^2 + y^2 = 144$ and

   From A, $x^2 + y^2 = 25$

   $\therefore 9y^2 + 25 = 144 \Rightarrow 9y^2 = 119$

   $\Rightarrow y = \pm\sqrt{\frac{119}{9}}$

   $\therefore x^2 + y^2 = 25 \Rightarrow x^2 = 25 = \frac{119}{9} = \frac{106}{9}$

   $\Rightarrow x = \pm\sqrt{\frac{106}{9}}$

   ∴ x has two value, y has two values

   ∴ possible ordered pairs = 4

$\therefore A \cap B$ has 4 elements

3. (a) 60 percent

   Explanation: Let A denote the set of persons travelling bu car, B denotes the set of persons travelling by bus, then

   $$n(A) = 20,\ n(B) = 50,\ n(A \cap B) = 10$$

   $$\therefore\ n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

   = 20 + 50 - 10 = 60

4. (c) $(-\infty, -2) \cup (2, \infty)$

   Explanation: For f(x) to be real, we much have,

   $$x^2 - 4 > 0$$

   $$\Rightarrow (x-2)(x+2) > 0$$

   $$\Rightarrow -\infty < -2 \ or\ 2 < x < \infty$$

   $\therefore$ Domain $= (-\infty, -2) \cup (2, \infty)$ (∞ means infinity)

5. (b) $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

   Explanation:

   gof(x)=g(f(x))

   $\Rightarrow gof(x) = g(\tan x) = \sqrt{3 - \tan^2 x}$

   which is defined only if

   $3 - \tan^2 x \geq 0$

   $\Rightarrow 3 \geq \tan^2 x$

   $\Rightarrow -\sqrt{3} \leq \tan x \leq \sqrt{3}$

   $\Rightarrow -\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

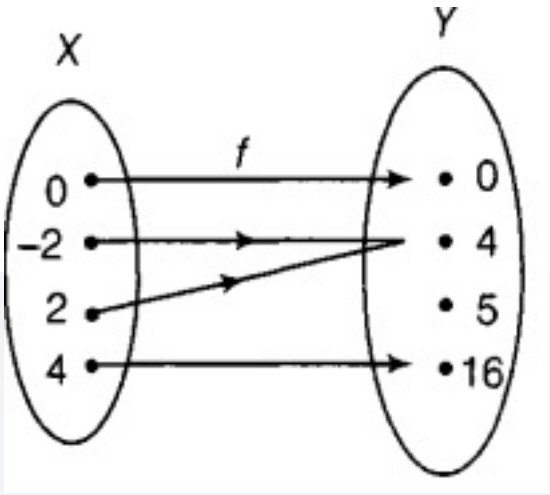
   so the domain of gof is $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$

6. domain

7. 6

8. R = {(x, x^3):x is a prime number less than 10}
   Putting x = 2, 3, 5 , 7
   R = {(2, 8), (3, 27), (5, 125), (7, 343)}

9. Domain of f = X = $\{0, \pm 2, 4\}$
   Range of f = $\{0, 4, 16\}$



10. Given, f: R $\rightarrow$ R and f(x) = x^2

    For all values of x, f(x) is positive or equal to zero.

    Thus, range of f is the set of all real numbers greater than or equal to zero.

    Range of f = {x : $x \geq 0,\ x\ \in\ R$}

11. Let (a, b) be an arbitrary element of A $\times$ A. Then,
    $(a, b) \in A \times A$
    $\Rightarrow \quad a \in A$ and $b \in A$
    $\Rightarrow \quad (a \in A, b \in A)$ and $(a \in A, b \in A)$
    $\Rightarrow \quad (a \in A, b \in B)$ and $(a \in B, b \in A)$ [Using $A \subseteq B$]
    $\Rightarrow \quad (a, b) \in (A \times B)$ and $(a, b) \in (B \times A)$
    $\Rightarrow \quad (a, b) \in (A \times B) \cap (B \times A)$
    $\therefore \quad A \times A \subseteq (A \times B) \cap (B \times A)$
    Hence, if $A \subseteq B \Rightarrow A \times A \subseteq (A \times B) \cap (B \times A)$

12. As we have, $f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$

For domain of $f(x)$

$4 - x \geq 0$ and $x^2 - 1 > 0$

$\Rightarrow \quad x \leq 4$ and $x^2 > 1$

$\Rightarrow \quad x \leq 4$ and $x \in (-\infty, -1) \cup (1, \infty)$

$\therefore \quad x \in (-\infty, -1) \cup (1, 4]$

13. According to the question, the relation between honest and punctual people is

$h = p + 16$

And the relation between progress and honest people is

$P = \left(\frac{h}{8}\right) + 5$

Required relation between the number of people who progress in life and punctual is given by,

$P = \left(\frac{p+16}{8}\right) + 5 [\because h = p + 16]$

P = $\left(\frac{p}{8}\right) + 2 + 5$

P = $\left(\frac{p}{8}\right) + 7$

We can complete our work on time and the quality of work will also good if we are punctual. This helps us to get progress in our life.

14. Here $f(x) = \frac{x^2-9}{x-3}$

f (x) assume real values for all real values of x except for x - 3 = 0 i.e .x = 3

Thus domain of f (x) = R - {3}

Let f (x) = y

$\therefore y = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{(x-3)}$

$\Rightarrow$ y = x + 3

y takes all real values except 6 as domain =R-{3}

Thus range of f (x) = R - {6}.

15. i.

    a. Let $(x, y) \in$ R

    $\Rightarrow x - y$ is divisible by n.

    $\Rightarrow x - y = kn$ for some k $\in$ Z

$\Rightarrow y - x = (-k)n$

$\Rightarrow y - x$ is divisible by n.

$\Rightarrow (y, x) \in R$

b. Let $(y, x) \in R$ and (y, z)$\in$ R

Now, $(y, x) \in R \Rightarrow x - y$ is divisible by n.

$\Rightarrow x - y = kn$ for some k $\in$ Z

Also, $(y, z) \in R \Rightarrow y - z$ is divisible by n.

$\Rightarrow y - z = mn$ for some m $\in$ Z.

$\Rightarrow (x - y) + (y - z) = kn + mn$

$\Rightarrow x - z = (k + m)n$

$\Rightarrow$ x - z is divisible by n.

$\Rightarrow$ (x, z) $\in R$

ii. Here, $f(x) = \frac{x^2-9}{x-3}$

f(x) assume all real values of x except for x - 3 = 0

i.e., x = 3.

Thus, domain of f(x) = R - {3}.

Let f(x) = y

$\therefore \quad y = \frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{(x-3)}$

$\Rightarrow y = x + 3$

Since y takes all real values except 6.

Thus, range of f(x) = R - {6}.

iii. Here, $f(x) = \frac{x^2+3x+5}{x^2+x-6}$

$= \frac{x^2+3x+5}{(x+3)(x-2)}$

The function f(x) is defined for all values of x except for $x + 3 = 0$ and $x - 2 = 0$

i.e., $x = -3$ and $x = 2$.

Thus, domain of f(x) = R - {-3, 2}.