CBSE Test Paper 01

CH-02 Relations and Functions

1. Two finite sets have m and n elements. The number o elements in the power set of the first is 48 more than the total number of elements in the power set of the second. Then the values of m and n are
   a. 6, 4
   b. 6, 3
   c. 3, 7
   d. 7, 6
2. Let $f\left(x+\frac{1}{x}\right)=x^2+\frac{1}{x^2}, x \neq 0,$ then f(x) =
   a. x^2-2
   b. x^2-1
   c. x^2
   d. x^2+1
3. The function f (x) = log $\left(x+\sqrt{x^2+1}\right)$ is
   a. a periodic function
   b. neither an even nor an odd function
   c. an odd function
   d. an even function
4. If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is
   a. 3^2
   b. 9^2
   c. 2^9-1
   d. 2^9
5. The relation R = {1, 1), (2, 2), (3, 3)} on the set {1, 2, 3) is
   a. an equivalence relation
   b. reflexive only
   c. symmetric only
   d. transitive only
6. If f(1 + x) = x^2 + 1, then f(2 - h) is ________.

7. Fill in the blanks: Let A and B be any two non-empty finite sets containing m and n elements respectively, then, the total number of subsets of (A $\times$ B) is ________.
8. If A $\times$ B = {(a, 1),(a, 5),(a, 2),(b, 2),(b, 5),(b, 1)}, then find A, B and B $\times$ A.
9. Find the domain of the function $f(x) = \frac{x^2+3x+5}{x^2+x-6}$.
10. Let f, g: $R \rightarrow R$ be defined, respectively by f(x) = x + 1, g(x) = 2x – 3. Find f + g, f – g and $\frac{f}{g}$.
11. If A = (1, 2, 3), B = {4}, C = {5}, then verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
12. Let f: R $\rightarrow$ R and g: C $\rightarrow$ C be two functions defined as f(x) = x^2 and g(x) = 2x. Are they equal functions?
13. If A = {2, 3}, B = {4, 5}, C = {5, 6}, find $A \times (B \cup C), A \times (B \cap C)$, $(A \times B) \cup (A \times C)$
14. If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$ find the values of x and y.
15. If A = {a,d}, B = {b, c, e} and C = {b, c, f}, then verify that
    i. A$\times$(B $\cup$ C) = (A$\times$B) $\cup$ (A$\times$C)
    ii. A$\times$(B $\cap$ C) = (A$\times$B) $\cap$ (A$\times$C)

Solution

1. (a) 6, 4

   Explanation:

   Let A has m elements and B gas n elements. Then, no. of elements in

   P(A) = 2^m and no. of elements in P(B) = 2^n.]

   By the question,

   $2^m = 2^n + 48$

   $\Rightarrow 2^m - 2^n = 48$

   This is possible, if $2^m = 64$, $2^n = 16$. (As 64 - 16 = 48)

   $\therefore 2^m = 64 \Rightarrow 2^m = 2^6$

   $\Rightarrow m = 6.$

   Also, $2^4 = 16 \Rightarrow 2^4 = 2^4$

   $\Rightarrow n = 4$

2. (a) $x^2 - 2$

   Explanation:

   $\mathrm{f}\left(\mathrm{x} + \frac{1}{\mathrm{x}}\right) = \mathrm{x}^2 + \frac{1}{\mathrm{x}^2} = \left(\mathrm{x} + \frac{1}{\mathrm{x}}\right)^2 - 2$

   $\therefore \mathrm{f}(\mathrm{x}) = \mathrm{x}^2 - 2$

3. (c) an odd function

   Explanation:

$$f(-x) = \log\left(-x + \sqrt{(-x)^2 + 1}\right) = \log\left(-x + \sqrt{x^2 + 1}\right)$$

$$= \log\left(\sqrt{x^2 + 1} - x\right) = \log\left(\frac{(\sqrt{x^2+1} - x)(\sqrt{x^2+1} + x)}{(\sqrt{x^2+1} + x)}\right)$$

$$= \log\left(\frac{1}{(\sqrt{x^2+1} + x)}\right) = \log(1) - \log\left(x + \sqrt{x^2 + 1}\right)$$

$$= 0 - \log\left(x + \sqrt{x^2 + 1}\right)$$

$$\Rightarrow f(-x) = -f(x)$$

$\Rightarrow$ f is an odd fucntion

4. (d) 2^9

   Explanation:

   Here, A = {2,3,4}; B={2,3,5}

   n(A) = 3, n)B) = 3

   $\therefore$no. of relations from A to B $= 2^{n(A)\times n(B)} = 2^{3\times 3} = 2^9$

5. (a) an equivalence relation

6. h^2 - 2h + 2

7. 2^{mn}

8. A $\times$ B = $\{(a,1),(a,5),(a,2),(b,2),(b,5),(b,1)\}$. Clearly,
   A is the set of first elements of all ordered pairs in A $\times$ B and B is set of second elements of all ordered pairs in A $\times$ B.
   $\therefore A =$ {a, b}, B = $\{1,5,2\}$
   and B $\times$ A = { 1,5,2}$\times${ a , b }
   = $\{(1,a),(1,b),(5,a),(5,b),(2,a),(2,b)\}$

9. Here $f(x) = \frac{x^2+3x+5}{x^2+x-6} = \frac{x^2+3x+5}{(x+3)(x-2)}$

The function f(x) is defined for all values of x except

x + 3 = 0, x - 2 = 0 i.e. x = -3 and x = 2

Thus domain of f(x) = R - {-3, 2}

10. Here f (x) = x + 1 and g (x) = 2x – 3

Now (f + g) (x) =f (x) + g(x) = x + 1 + 2x - 3 = 3x - 2

(f - g) (x) = f(x) - g(x) = x + 1- (2x - 3) = x + 1 - 2x + 3 = -x + 4

$\frac{(f)}{(g)}(x) = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$

11. As given in the question,

A = {1, 2, 3}, B = {4} and C = {5}

$\therefore \quad B \cup C = \{4\} \cup \{5\}$ = {4, 5}

$\therefore \quad A \times (B \cup C)$= {1, 2, 3} $\times$ {4, 5}

$\Rightarrow \quad A \times (B \cup C)$= {(1, 4), (1, 5), (2, 4) , (2, 5), (3, 4), (3, 5)}.......(a)

Now,

$(A \times B)$ = {1, 2, 3} $\times$ {4} = {(1, 4), (2, 4), (3, 4)}

and, $(A \times C)$ = {1, 2, 3}$\times$ {5} = {(1, 5), (2, 5), (3, 5)}

$\therefore \quad (A \times B) \cup (A \times C)$= {(1, 4), (2, 4), (3, 4)} $\cup$ {(1, 5), (2, 5), (3, 5)}

$\Rightarrow \quad (A \times B) \cup (A \times C)$= {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)}......(b)

From equations (a) and (b), we get

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence verified.

12. We have,

f : R $\rightarrow$ R and g : C $\rightarrow$ C

where R is set of Real numbers and C is set of Complex numbers.

From definitions as given,

Domain of f = R and

Domain of g = C

Now, Two functions are said to be equal when domain and co-domain of both the functions are equal.

As, Domain of f ≠ Domain of g,
∴ f(x) and g(x) are not equal functions.

13. We have,
A = {2, 3}, B = {4, 5}, C = {5, 6}
$\therefore \quad B \cup C = \{4, 5\} \cup \{5, 6\}$
= {4, 5, 6}
$\therefore \quad A \times (B \cup C) = \{2, 3\} \times \{4, 5, 6\}$
= {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}
Now,
$B \cap C = \{4, 5\} \cap \{5, 6\} = \{5\}$
$\therefore \quad A \times (B \cap C) = \{2, 3\} \times \{5\}$
={(2, 5), (3, 5)}
Now,
$A \times B = \{2, 3\} \times \{4, 5\}$
={(2, 4), (2, 5), (3, 4),(3, 5)}
and, $A \times C = \{2, 3\} \times \{5, 6\}$
= {(2, 5), (2, 6), (3, 5), (3, 6)}
$\therefore \quad (A \times B) \cup (A \times C)$= {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

14. Here $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$
$\therefore \frac{x}{3} + 1 = \frac{5}{3}$ and $y - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \frac{x}{3} = \frac{5}{3} - 1$ and $y = \frac{1}{3} + \frac{2}{3}$
$\Rightarrow \frac{x}{3} = \frac{2}{3}$ and $y = \frac{3}{3}$
$\Rightarrow$ x = 2 and y = 1

15. i. **To determine A $\times$ (B $\cup$ C)**

$B \cup C = \{b, c, e\} \cup \{b, c, f\} = \{b, c, e, f\}$

$\therefore A \times (B \cup C) = \{a, d\} \times \{b, c, e, f\}$

$= \{(a, b), (a, c), (a, e), (a, f), (d, b), (d, c), (d, e), (d, f)\}$ ...(i)

To determine $(A \times B) \cup (A \times C)$

$A \times B = \{a, d\} \times \{b, c, e\}$

$= \{(a, b), (a, c), (a, e), (d, b), (d, c), (d, e)\}$

$A \times C = \{a, d\} \times \{b, c, f\}$

$= \{(a, b), (a, c), (a, f), (d, b), (d, c), (d, f)\}$

$\therefore (A \times B) \cup (A \times C)$

$= \{(a, b), (a, c), (a, e), (a, f), (d, b), (d,c), (d,e),(d,f)\}$ ...(ii)

From Eqs. (i) and (ii), we get

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence verified.

ii. **To determine $A \times (B \cap C)$**

$(B \cap C) = \{b, c, e\} \cap \{b, c, f\} = \{b, c\}$

$\therefore A \times (B \cap C) = \{a, d\} \times \{b, c\}$

$= \{(a, b), (a, c), (d, b), (d, c)\}$ ...(iii)

To determine $(A \times B) \cap (A \times C)$

$A \times B = \{(a, b), (a, c), (a, e), (d, b), (d, c), (d, e)\}$

$A \times C = \{(a, b), (a, c), (a, f), (d, b), (d, c), (d, f)\}$

$\therefore (A \times B) \cap (A \times C) = \{(a, b), (a, c), (d, b), (d, c)\}$ ...(iv)

From Eqs. (iii) and (iv), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence verified.