CBSE Test Paper 02

CH-15 Statistics

1. The H.M. of 4,8,16 is
   a. 6.7
   b. 6.85
   c. 7.8
   d. 6.4
2. If in moderately asymmetrical distribution mode and mean of the data are 6 μ and 9 μ respectively, then median is
   a. 8 μ
   b. 7 μ
   c. 6 μ
   d. 5 μ
3. If the two lines of regression are at right angles , then $\rho(X,Y)$ is equal to
   a. 1
   b. 0
   c. - 1
   d. 1 or - 1
4. The mean weight of a group of 10 items is 28 and that of another group of n items is 35.The mean of combined group of 10 + n items is found to be 30. The value of n is
   a. 12
   b. 4

c. 2

d. 10

5. Product of n positive numbers is unity. The sum of these numbers cannot be less than

a. n^3

b. n

c. 1

d. n^2

6. Fill in the blanks:

For a sample of size 60, if $\sum x_i^2$ = 18000 and $\sum x_i$ = 960, then the variance is ________.

7. Fill in the blanks:

The mean of 100 observations is 50 and their standard deviation is 5. The sum of all squares of all the observations is ________.

8. The scores of a batsman in 10 matches were as follows: 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. Compute the variance and standard deviation.

9. Compute the mean deviation from the median of the following distribution:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	5	10	20	5	10

10. Calculate the mean deviation about the median of the observations: 38, 70, 48, 34, 63, 42, 55, 44, 53, 47.

11. The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12, 14 find the remaining two observations.

12. Find the mean deviation about the mean for the data

	0-	100-	200-	300-	400-	500-	600-	700-

Income per day	100	200	300	400	500	600	700	800
Number of persons	4	8	9	10	7	5	4	3

13. Find the mean deviation about the mean for the following data.

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

14. Find the mean deviation about the median for the data in: 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

15. The Arithmetic Mean, AM and Standard Deviation, SD of 100 items was recorded as 40 and 5.1, respectively. Later on, it was discovered that one observation 40 was wrongly copied down as 50. Find the correct SD.

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Solution

1. (b) 6.85

   Explanation:

   $$H.M = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}} = \frac{3}{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}} = 48/7 = 6.85$$

2. (a) 8 μ

   Explanation:

   $$median = \frac{\bmod e + 2mean}{3} = \frac{6\mu+2(9\mu)}{3} = \frac{24\mu}{3} = 8\mu$$

3. (b) 0

   Explanation: $\tan\theta = \frac{1-\rho^2}{\rho} \cdot \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$

   $\Rightarrow \tan\frac{\pi}{2} = \frac{1-\rho^2}{\rho} \cdot \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$

   $\Rightarrow \infty = \frac{1-\rho^2}{\rho} \cdot \frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$

   $\Rightarrow \rho = 0$

4. (b) 4

   Explanation: sum of weights of 10 items =280

   sum of weights of n items = 35n

   so, sum of weights of (10 + n) items = 280 + 35n

   so ,mean = (280 + 35 n) / (10 + n)

   30(10 + n) = 280 + 35n

   solving we get, n = 4

5. (b) n

   Explanation:

we know that, for a set of observations $A.M. > G.M.$

if $X_1, X_2, X_3, \ldots, X_n$ are the observations then

$(X_1 + X_2 + X_3 + \ldots\ldots\ldots + X_n)/n \succ \sqrt{X_1 . X_2 . X_3 \ldots X_n}$

as $(X_1 . X_2 . X_3 \ldots X_n) = 1$

therefore, $(X_1 + X_2 + X_3 + \ldots + X_n)/n \succ 1$

$\Rightarrow (X_1 + X_2 + X_3 + \ldots + X_n) \succ n$

6. 44

7. 252500

8. Let the assumed mean be A = 48.
   Calculation of Variance:

x_i	$d_i = x_i - A = x_i - 48$	d_i^2
38	-10	100
70	22	484
48	0	0
34	-14	196
42	-6	36
55	7	49
63	15	225
46	-2	4
54	6	36
44	-4	16
	$\Sigma d_i = 14$	$\Sigma d_i^2 = 1146$

Here, n = 10, $\Sigma d_i = 14$ and $\Sigma d_i^2 = 1146$

$$\therefore \quad \text{Var}(x) = \frac{1}{n}\left(\Sigma d_i^2\right) - \left(\frac{1}{n}\Sigma d_i\right)^2 = \frac{1146}{10} - \left(\frac{14}{10}\right)^2 = 112.64$$

Hence, $S.D. = \sqrt{\text{Var}(x)} = \sqrt{112.64} = 10.61$

9. We have to calculate mean deviation from the median. So, first we calculate the median with the help of following table,

Class	x_i	f_i	CF	$\lvert d_i\rvert = \lvert x_i - 25\rvert$	$f_i d_i$
0-10	5	5	5	20	100
10-20	15	10	15	10	100
20-30	25	20	35	0	0
30-40	35	5	40	10	50
40-50	45	10	50	20	200
		$\sum f_i 50$			$\sum f_i d_i 450$

N/2 = 50/2 = 25, so median class is 20 - 30.

Now, we will use following formula to calculate median for the given data,

$$Median = l + \frac{C.F - N/2}{f} \times h = 20 + \frac{35-25}{20} \times 10$$

Thus, median = 25

$$M.D. = \frac{1}{n}\sum f_i \lvert d_i \rvert = \frac{1}{50}[450] = 9$$

10. To find the median arrange the observations in ascending order of magnitude, we will get

34, 38, 42, 44, 47, 48, 53, 55, 63, 70

Clearly, the middle observations are 5^{th} and 6^{th} observations, which are 47 and 48. So, median = 47.5 (middle value of 47 and 48)

To calculate mean deviation, we have to find d_i as follows:

x_i	$d_i = \lvert x_i - \text{median}\rvert$
38	9.5
70	22.5

48	0.5
34	13.5
63	15.5
42	5.5
55	7.5
44	3.5
53	5.5
47	0.5
Total	84

We have,

$\sum|x_i - 47.5| = \sum d_i = 84$

$\therefore \quad M.D. = \frac{1}{n}\Sigma|d_i| = \frac{1}{10}[84] = 8.4$

11. Let two remaining observations be x and y. Then

$\frac{2+4+10+12+14+x+y}{7} = 8$

$\therefore$ 42 + x + y = 56 $\Rightarrow$ x + y = 14

Also $\frac{1}{7}$ $(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - (8)^2 = 16$

$\Rightarrow \frac{1}{7}(4 + 16 + 100 + 144 + 196 + x^2 + y^2) - 64 = 16$

$\Rightarrow 460 + x^2 + y^2 = 560 \Rightarrow x^2 + y^2 = 100$ .... (ii)

Now $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$\Rightarrow (14)^2 + (x - y)^2 = 2 \times 100$

$\Rightarrow (x - y)^2 = 200 - 196 \Rightarrow (x - y)^2 = 4 \Rightarrow x - y = \pm 2$

When x - y = 2

Solving x + y = 14 and x - y = 2 we get x = 8 and y = 6

When x - y = -2

Solving x + y = 14 and x - y = - 2 we get x = 6 and y = 8

12.

Income per day	Mid values x_i	f_i	f_ix_i	$\lvert x_i - 358\rvert$	$f_i\lvert x_i - 358\rvert$

0 - 100	50	4	200	308	1232
100 - 200	150	8	1200	208	1664
200 - 300	250	9	2250	108	972
300 - 400	350	10	3500	8	80
400 - 500	450	7	3150	92	644
500 - 600	550	5	2750	192	960
600 - 700	650	4	2600	292	1168
700 - 800	750	3	2250	392	1176
		50	17900		7896

Mean $\bar{x} = \frac{1}{N} \sum f_i x_i = \frac{1}{50} \times 17900 = 358$

Mean deviation about mean $= \frac{1}{N} \sum_{i=1}^{n} f_i \left| x_i - \bar{x} \right|$

$= \frac{1}{50} \times 7896 = 157.92$

13. We make the table from the given data.

x_i	f_i	$f_i x_i$	$\lvert x_i - \overline{x} \rvert$	$f_i \lvert x_i - \overline{x} \rvert$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
	$\sum f_i$=40	$\sum f_i x_i$=300		$\sum f_i \lvert x_i - \overline{x} \rvert$=92

Here, N = $\sum f_i$ = 40, $\sum f_i x_i$ = 300 and $\sum f_i \left| x_i - \overline{x} \right|$ = 92

Now, mean($\overline{x}$) = $\frac{1}{N} \sum f_i x_i$ = $\frac{1}{40}$ $\times$ 300 = 7.5

$\therefore$ Mean deviation about the mean, MD($\overline{x}$) = $\frac{1}{N}\sum f_i\left|x_i-\overline{x}\right|$ = $\frac{1}{40}$ $\times$ 92 = 2.3

Hence, the mean deviation about mean is 2.3

14. Arrange the data in ascending order, we have

    10, 11, 11,12, 13, 13, 14, 16, 16, 17, 17, 18

    Here n = 12 (which is even)

    So median is average of 6 $^{\text{th}}$ and 7 $^{\text{th}}$ observations

    $\therefore$ Median $=\frac{13+14}{2}=\frac{27}{2}$ = 13.5

x_i	$\lvert x_i - M\rvert$
10	3.5
11	2.5
11	2.5
12	1.5
13	0.5
13	0.5
14	0.5
16	2.5
16	2.5
17	3.5
17	3.5
18	4.5
Total	28

M.D. about median $=\frac{1}{n}\sum_{i=1}^{n}\left|x_i-M\right|$

$=\frac{1}{12}\times 28=2.33$

15. Number of items = 100

    Incorrect mean ($\overline{x}$) = 40

Incorrect SD = 5.1

Incorrect item = 50

Correct item = 40

Now, $\overline{x} = \frac{\sum x}{n}$ $\Rightarrow$ 40 = $\frac{\sum x}{100}$

$\Rightarrow$ Incorrect $\sum x$ = 4000

$\Rightarrow$ Correct $\sum x = 4000 - 50 + 40 = 3990$

$\therefore$ Correct mean = $\frac{3990}{100}$ = 39.9

Now, Incorrect SD = $\sqrt{\frac{\sum(x_i - \overline{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - (\overline{x})^2}$

$\Rightarrow 5.1 = \sqrt{\frac{\text{Incorrect } \sum x^2}{100} - (40)^2}$

$\Rightarrow 26.01 = \frac{\text{Incorrect } \sum x^2}{100} - 1600$

$\therefore$ Incorrect $\sum x^2$ = 162601

Now, correct $\sum x^2 = 162601 - (50)^2 + (40)^2$ = 161701

and Correct SD = $\sqrt{\frac{161701}{100} - (39.9)^2}$ = $\sqrt{1617.01 - 1592.01} = 5$