CBSE Test Paper 02

CH-14 Mathematical Reasoning

1. Let p and q be two propositions. Then the contrapositive of the implication $p \rightarrow q$ is
   a. $\sim q \rightarrow p$
   b. $\sim q \rightarrow \sim p$
   c. $p \leftrightarrow q$
   d. $\sim p \rightarrow \sim q$
2. The proposition $p \rightarrow \sim (p \wedge \sim q)$ is
   a. neither a contradiction nor a tautology
   b. a tautology
   c. a contradiction
   d. none of these
3. Which of the following proposition is a tautology ?
   a. $\sim p \wedge (\sim p \vee \sim q)$
   b. $\sim q \wedge (\sim p \vee \sim q)$
   c. $(\sim p \vee \sim q) \wedge (p \vee \sim q)$
   d. $(\sim p \vee \sim q) \vee (p \vee \sim q)$
4. Consider the proposition: "If the pressure increases, then volume decreases ". Negative of this proposition is
   a. The pressure increases but volume does not decrease.
   b. If the volume increases, the pressure decreases

c. If the pressure does not increase the volume does not decrease

d. If the volume decreases, then the pressure increases

5. Which of the following pairs is logically equivalent ?

   a. Inverse , Contrapositive

   b. Conditional , Contrapositive

   c. Conditional , Inverse

   d. Contrapositive , Converse

6. Fill in the blanks:

   The quantifier in the statement "There exists a number which is a multiple of 6 and 9" is ________.

7. Fill in the blanks:

   The quantifier in the statement "For all even integers x, x^2 is also even" is ________.

8. Find out below sentence is a statement or not. justify your answer.
   Every set is a finite set.

9. Write the component statements of the given compound statement and check whether the compound statement is true or false:
   Square of an integer is positive or negative.

10. Write the negation of the statement:
    There exists $x \in N, x + 3 = 10$

11. Write the negation of the following statements.

    i. Paris is in France and London is in England.
    ii. $2 + 3 = 5$ and $8 < 10$.

12. Show that the following statement is true by using contrapositive method:
    'If x, y are integers such that xy is odd, then both x and y are odd integers'.

13. Show that the following statement is true by the method of contrapositive
p : "If x is an integer and $x \in Z$ is even, then x is also even"

14. For each of the following compound statement first identify the connecting words and then break it into component statements.
(i) All rational numbers are real and all real numbers are not complex.
(ii) Square of an integer is positive or negative.
(iii) The sand heats up quickly in the sun and does not cool down fast at night.
(iv) x = 2 and x = 3 are the roots of the equation $3x^2 - x - 10 = 0$

15. Write the negation of the following statements:
(i) p : For every positive real number x, the number x - 1 is also positive.
(ii) q : All cats scratch.
(iii) r : For every real number x, either $x > 1$ or $x < 1$.
(iv) s : There exists a number x such that $0 < x < 1$.

Solution

1. (b) $\sim q \rightarrow \sim p$

   Explanation: The contrapositive of $p \rightarrow q \equiv \sim q \rightarrow \sim p$
2. (a) neither a contradiction nor a tautology

   Explanation:

p	q	$\sim q$	$\sim (p \wedge \sim q)$	$p \rightarrow (p \wedge \sim q)$
T	T	F	T	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

   since propositiion has both T and F.hence option a
3. (d) $(\sim p \vee \sim q) \vee (p \vee \sim q)$

   Explanation: $(\sim p \vee p) \equiv T$ and $\sim q \vee q \equiv T$ Associative law $T \vee T \equiv T$
4. (a) The pressure increases but volume does not decrease.

   Explanation: p:The pressure increases; q:Volume decreases. $\sim (p \rightarrow q) \equiv p \wedge \sim q$
5. (b) Conditional , Contrapositive

   Explanation: conditional $p \rightarrow q \equiv \sim p \vee q$ contrapositive $\sim q \rightarrow \sim p \equiv q \vee \sim p$

   hence conditional and contrapositive are equal
6. there exists
7. for all
8. A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.

   This sentence is always false because there are sets which are not finite. Hence, it is a statement.
9. The component statements of the given compound statement : Square of an integer is positive or negative.

   p: Square of an integer is positive.

   q: Square of an integer is negative.

Hence, the compound statement is false because p is true and q is false.

10. The negation of the statement

    There exists x $\in$ N, x + 3 = 10

    For every x $\in$ N, x + 3 $\neq$ 10

11. i. Let p : Paris is in France and q : London is in England.

       Then, the conjunction is $p \wedge q$.

       Now, $\sim$ p : Paris is not in France,

       and, $\sim$ q : London is not in England.

       So, negation of $p \wedge q$ is given by

       $\sim (p \wedge q) = \sim p \vee \sim q$

       = Paris is not in France or London is not in England.

    ii. Let p : 2 + 3 = 5, q : 8 < 10

       Then, the conjunction is $p \wedge q$ is

       $\sim (p \wedge q) = \sim p \vee -q = (2 + 3 \neq 5)$ or $(8 \nless 10)$

12. Let p and q be two statements given by

    p : xy is an odd integer.

    q : Both x and y are odd integers.

    Let q be not true. Then, q is not true i.e., it is false that both x and y are odd integers.

    So, at least one of x and y is an even integer.

    Let, x = 2n for some integer n.

    Then, xy = 2ny for some integer n.

    $\therefore$ xy is an even integer. i.e., $\sim$ p is true.

    Thus, $\sim q \Rightarrow \sim p$

    Hence, the given statement is true.

13. The given compound statement is of the form "if p then q"

    p: $x \in Z$ and x^2 is even

    q : x is an even integer.

    We assume that q is false then x is not an even integer.

    $\Rightarrow$ x is an odd integer

    $\Rightarrow$ x^2 is an odd integer.

    $\Rightarrow$ P is false

    So when q is false, p is false.

Thus the given compound statement is true.

14. (i) The component statement has the connecting word 'and' .component statements are

p : All rational numbers are real.

q : All real numbers are not complex.

(ii) The component statement has the connecting words 'or' .component statements are

p : Square of an integer is positive.

q : Square of an integer is negative.

(iii) The component statement has the connecting word 'and' .component statements are

p : The sand heats up quickly in the sun.

q : The sand does not cool down fast at night.

(iv) The component statement has the connecting word 'and' component statements are

p : $x = 2$ is a root of the equation $3x^2 - x - 10 = 0$

q : $x = 3$ is a root of the equation $3x^2 - x - 10 = 0$

15. (i) ~ p : There exists a positive real number x such that x - 1 is not positive.

(ii) ~ q : There exists a cat which does not scratch.

(iii) ~ r : There exists a real number x such that neither $x > 1$ nor $x < 1$.

(iv) ~ s : There does not exist a number x such that $0 < x < 1$.