CBSE Test Paper 01

CH-14 Mathematical Reasoning

1. The proposition $(p \to \sim p) \wedge (\sim p \to p)$ is
   a. a contradiction and a tautology
   b. a tautology
   c. neither a contradiction nor a tautology
   d. a contradiction
2. Which of the following is a proposition ?
   a. A half open door is half closed
   b. A triangle is a circle and 10 is a prime number
   c. Logic is an interesting subject
   d. I am a lion
3. Let p and q be two propositions. Then the inverse of the implication $p \to q$ is
   a. $\sim p \to q$
   b. $\sim q \to p$
   c. $p \to \sim q$
   d. $\sim p \to \sim q$
4. $\sim (p \wedge q)$ is logically equivalent to
   a. $\sim p \to q$
   b. $\sim p \leftrightarrow \sim q$
   c. $\sim p \to \sim q$
   d. $\sim p \vee \sim q$
5. $(p \wedge \sim q) \wedge (\sim p \vee q)$ is
   a. a contradiction
   b. both a tautology and a contradiction
   c. neither a tautology nor a contradiction
   d. a tautology
6. Fill in the blanks:

   Truth and falsity of a statement is called its ________.

7. Fill in the blanks:

The negation of a disjunction $p \vee q$ is the ________ of the negation of p and the negation of q.

8. Write the negation of the statement:
   For every $x \in N$, $x + 3 < 10$

9. Find out below sentence is a statement or not. justify your answer.
   Are all circles round?

10. Determine whether the statement is an inclusive OR or exclusive OR. Give reasons for your answer.
    Students can take Hindi or Sanskrit as their third language.

11. Write the component statements of the compound statements and check whether the compound statement is true or false:
    All rational numbers are real and all real numbers are not complex.

12. Which of the following statements are compound statements?

    i. 2 is both an even and a prime number.
    ii. 9 is neither an even number nor a prime number.

13. Verify by the method of contradiction that $\sqrt{7}$ is irrational.

14. Verify by method of contradiction, $\sqrt{11}$ is irrational.

15. Write the contra positive and converse of the following statements.
    (i) If x is a prime number, then x is odd.
    (ii) If the two lines are parallel, then they do not intersect in the same plane.
    (iii) Something is cold implies that it has low temperature.
    (iv) You cannot comprehend geometry if you do not know how to reason deductively.
    (v) x is an even number implies that x is divisible by 4.

Solution

1. (d) a contradiction
   Explanation: it can be simplified as $p \leftrightarrow \sim p$ since $(p \rightarrow q) \wedge (q \rightarrow p) \equiv p \leftrightarrow q$ F ($T \leftrightarrow F \equiv F$ and $F \leftrightarrow T \equiv F$)
2. (b) A triangle is a circle and 10 is a prime number
   Explanation: it is a statement which is F.Hence it is a proposition. Other options are open sentences which are not propositions
3. (d) $\sim p \rightarrow \sim q$
   Explanation: $inverse\ of\ p \rightarrow q \equiv \sim p \rightarrow \sim q$
4. (d) $\sim p \vee \sim q$
   Explanation: $\sim (p \wedge q) \equiv \sim p \vee \sim q$ De Morgan's law
5. (a) a contradiction
   Explanation: $[(p \wedge \sim q) \wedge (\sim p)] \vee [(p \wedge \sim q) \wedge q)]$ Since $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ F V F = F Since $p \wedge \sim p = F$ Hence contradiction
6. truth value
7. conjunction
8. Given :For every x ∈ N, x + 3 < 10
   The negation of the statement:
   x ∈ N such that x + 3 $\geq$ 10.
9. A statement or a proposition is an assertive (or a declarative) sentence which is either true or false but not both.
   The sentences "Are all circles round?" is an interrogative sentence. So, it is not a statement.
10. The statement is, "Students can take Hindi or Sanskrit as their third language". An exclusive "OR" is used because a student cannot take both Hindi and Sanskrit as the third language.
11. The component statements of the compound statement.
    All rational numbers are real and all real numbers are not complex.
    p: All rational numbers are real.

q: All real numbers are not complex.

we know that p is true and q is false.

$\therefore$ The compound statement "p and q" is false.

12. i. The given statement can be broken into two simple statements.

    p: 2 is an even number.

    q: 2 is a prime number.

    The above two simple statements are connected by the connective and. Hence, it is a compound statement.

    ii. The given statement can be broken into two simple statements.

    p: 9 is not an even number

    q: 9 is not a prime number.

    The above two simple statements are connected by the connective 'or'. Hence, it is a compound statement.

13. Let p be the statement given by p: $\sqrt{7}$ is irrational.

    If possible, let p be not true i.e. let p be false. Then,

    p is false.

    $\Rightarrow \sqrt{7}$ is rational

    $\Rightarrow \sqrt{7} = \frac{a}{b}$, where a and b are integers having no common factor.

    $\Rightarrow 7 = \frac{a^2}{b^2}$

    $\Rightarrow a^2 = 7b^2$

    $\Rightarrow$ 7 divides a^2

    $\Rightarrow$ 7 divides a

    $\Rightarrow$ a = 7 c for some integer c

    $\Rightarrow a^2 = 49c^2$

    $\Rightarrow 7b^2 = 49c^2$ [$\because a^2 = 7b^2$]

    $\Rightarrow b^2 = 7c^2$

    $\Rightarrow$ 7 divides b^2

    $\Rightarrow$ 7 divides b

    Thus, 7 is a common factor of both a and b. This contradicts that a and b have no common factor. So, the supposition $\sqrt{7}$ is rational is wrong. Hence, the statement "$\sqrt{7}$ is irrational" is true.

14. Let the given statement be false.

i.e., $\sqrt{11}$ is rational.

It means $\sqrt{11} = \frac{p}{q}$, where p and q are prime.

On squaring both sides, we get

$11 = \frac{p^2}{q^2} \Rightarrow p^2 = 11q^2$ ..... (i)

It means 11 divides p. ..... (ii)

Thus, there exists an integer r such that

$p = 11r \Rightarrow p^2 = 121r^2$ ...... (iii)

From Eqs. (i) and (iii), we get

$11q^2 = 121r^2 \Rightarrow q^2 = 11r^2$

It means 11 divides q. .... (iv)

From Eqs. (ii) and (iv), we get

11 divides p and q.

It means 11 is a common factor of p and q which contradict our assumption that p and q have no common factor.

Hence, $\sqrt{11}$ is rational is false and $\sqrt{11}$ is irrational.

15. (i) Here p: x is a prime number.

q: x is odd.

Now ~p: x is not a prime number.

~q: x is not odd.

The contrapositive of given statement is:

If x is not odd then x is not a prime number.

The converse of given statement is:

If x is an odd number then x is a prime number.

(ii) Here p: Two lines are parallel.

q: They do not intersect in the same plane.

Now ~p: Two line are not parallel.

~q: They intersect in the same plane.

The contrapositive of given statement is:

If two lines intersect in the same plane then they are not parallel.

The converse of given statement is:

If the two lines do not intersect in the same plane then they are parallel

(iii) Here p: Something is cold.

q: It has low temperature.

Now ~ p: Something is not cold.

~q: It has not low temperature.

The contrapositive of given statement is:

If something does not have low temperature then it is not cold.

The converse of given statement is:

If something has low temperature then it is cold.

(iv) Here p: You can not comprehend geometry:

q: You do not know how to reason deductively.

Now ~p: You can comprehend geometry.

~q: You know how to reason deductively.

The contrapositive of given statement is:

If you know to reason deductively then you can comprehend geometry.

The converse of given statement is:

If you do not know how to reason deductively then you can not comprehend geometry.

(v) Here p: x is an even number.

q: x is divisible by 4.

Now ~p: x is not an even number.

- q: x is not divisible by 4.

The contrapositive of given statement is:

If x is not divisible by 4 then x is not an even number.

The converse of given statement is:

If x is divisible by 4 then x is an even number.