CBSE Test Paper 02

CH-13 Limits and Derivatives

1. $\underset{x \to 0}{Lt} \frac{x \log(1+x)}{1-\cos x}$ is equal to

   a. 1

   b. 2

   c. $\frac{1}{2}$

   d. 0

2. If y = log x , then $y_n =$

   a. $\frac{(-1)^n . n!}{x^n}$.

   b. $\frac{(-1)^n . n!}{x^{n+1}}$

   c. $\frac{(-1)^{n-1} (n-1)!}{x^n}$

   d. $\frac{(-1)^n (n-1)!}{x^n}$

3. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ holds true for

   a. all real x

   b. all x [- 1, 1]

   c. all x $\in$ (- 1, 1)

   d. all real x for which | x| > 1

4. $\underset{x \to 0}{Lt} \frac{1-\cos x}{\sqrt{1+x}-1}$ is equal to

   a. 0

   b. $\sqrt{2}$

c. 1

d. $\frac{1}{2}$

5. If $y = \frac{1-x}{1+x} then\ y_6 =$

a. $\frac{6!}{(1+x)^7}$

b. $\frac{(-2).(7!)}{(1+x)^6}$

c. $\frac{(-2).(6!)}{(1+x)^7}$

d. $\frac{(2).(6!)}{(1+x)^7}$

6. Fill in the blanks:

The derivative of 5 secx + 4 cosx is ________.

7. Fill in the blanks:

The derivative of $2x - \frac{3}{4}$ is ________.

8. Find the value of $\lim\limits_{x \to \sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2}x-8}$.

9. Evaluate $\lim\limits_{x \to 2} \frac{x^7-128}{x-2}$.

10. Find the derivative of the following functions: cosec x

11. Evaluate $\lim\limits_{x \to a} \frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$.

12. Find the derivative of f(x) = 1 + x + x^2 + x^3 + ... + x^{50} at x = 1.

13. If $f(x) = \begin{cases} \frac{x-|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$, then show that $\lim\limits_{x \to 0}$ f(x) does not exist.

14. Find the derivative of the following functions: 3 cot x + 5 cosec x

15. Find the derivative of (sinx + cosx) from first principle.

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Solution

1. (b) 2

   Explanation: $\underset{x \to 0}{Lt} \frac{x \log(1+x)}{1-\cos x} \cdot \frac{x}{x}$

   $\Rightarrow \underset{x \to 0}{\text{Lt}} \left(\frac{\log(1+x)}{x}\right) \cdot \left(\frac{x^2}{1-\cos x}\right)$

   $\Rightarrow 1. \frac{1}{\frac{1}{2}}$

   $\Rightarrow 2$

2. (c) $\frac{(-1)^{n-1}(n-1)!}{x^n}$

   Explanation: y = log x

   $y' = \frac{1}{x}$

   $y'' = -\frac{1}{x^2}$

   $y''' = \frac{1.2}{x^3}$

   $y'''' = -\frac{1.2.3}{x^4}$

   .

   .

   $.y'^n = \frac{(-1)^{n-1}(n-1)!}{x^n}$

3. (c) all x $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ (- 1, 1)

   Explanation: Domain of $\sin^{-1} x = x \in [-1, 1]$ ....(1)

   domain of $\frac{1}{\sqrt{(1-x^2)}} = x \in (-1, 1)$ ....(2)

   Intersection of (1) and (2) will give (-1,1)

4. (a) 0

   Explanation: Since when we put x=0, we get 0/0 form, So we have to use D'L hospital rule:

   Do the differentiation of numerator and denominator partially w.r.t x, we get :

$\frac{\sin x}{\frac{1}{2\sqrt{1+x}}}$

By putting x=0 directly, we get 0.

5. (d) $\frac{(2).(6!)}{(1+x)^7}$

Explanation:

$y_1 = \frac{-2}{(1+x)^2}$

$y_2 = \frac{(-2)(-2)}{(1+x)^3}$

$y_3 = \frac{(-2)(-2)(-3)}{(1+x)^4}$

$y_4 = \frac{(-2)(-2)(-3)(-4)}{(1+x)^5}$

$y_5 = \frac{(-2)(-2)(-3)(-4)(-5)}{(1+x)^6}$

$y_6 = \frac{(-2)(-2)(-3)(-4)(-5)(-6)}{(1+x)^7}$

6. 5 secx tanx - 4 sinx

7. 2

8. $\lim_{x\to\sqrt{2}} \frac{x^4-4}{x^2+3\sqrt{2}x-8} = \lim_{x\to\sqrt{2}} \frac{(x+\sqrt{2})(x-\sqrt{2})\left(x^2+2\right)}{(x-\sqrt{2})(x+4\sqrt{2})}$
$= \lim_{x\to\sqrt{2}} \frac{(x+\sqrt{2})\left(x^2+2\right)}{(x+4\sqrt{2})}$
$= \frac{(\sqrt{2}+\sqrt{2})(2+2)}{(\sqrt{2}+4\sqrt{2})} = \frac{8\sqrt{2}}{5\sqrt{2}} = \frac{8}{5}$

9. $\lim_{x\to 2} \frac{x^7-128}{x-2} = \lim_{x\to 2} \frac{x^7-2^7}{x-2}$
$= 7(2)^{7-1} \left[\because \lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}\right]$
$= 7 \times 2^6$
$= 7 \times 64$
$= 448$

10. Here f (x) = cosec x

$\therefore$ f'$(x) = \frac{d}{dx}(\operatorname{cosec} x)$

= - cosec x cot x

11. $\lim\limits_{x \to a} \frac{(x+2)^{3/2}-(a+2)^{3/2}}{x-a}$

$= \lim\limits_{(x+2) \to (a+2)} \frac{(x+2)^{3/2}-(a+2)^{3/2}}{(x+2)-(a+2)} \left[\begin{array}{l}\text{as } x \to a \\ \therefore x+2 \to a+2\end{array}\right]$

$= \frac{3}{2}(a+2)^{\frac{3}{2}-1}\left[\because \lim\limits_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$

$= \frac{3}{2}(a+2)^{1/2}$

12. Given, f (a) = 1+ x + x^2 + x^3 +... + x^{50}

On differentiating both sides w.r.t. x, we get

f'(x) = 0 + 1 + 2x + $3x^2$ +... + $50x^{49}$

At x = 1,

f'(1) = 1 + 2(1)+ $3(1)^2$ + ... + $50(1)^{49}$

= 1 + 2 + 3 + ... + 50

$= \frac{(50)(51)}{2} = 1275\left[\because \sum_n = \frac{n(n+1)}{2}\right]$

13. Given, $f(x) = \begin{cases} \frac{x-|x|}{x}, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$

Now, LHL of f(x) at x = 0

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \frac{x-|x|}{x} = \lim\limits_{h \to 0^-} \frac{(0-h)-|0-h|}{(0-h)}$

[putting x = 0 - h as x $\to$ 0, then h $\to$ 0]

$= \lim\limits_{h \to 0^-} \frac{-h-h}{-h} = \lim\limits_{h \to 0} \frac{-2h}{-h} = \lim\limits_{h \to 0} 2 = 2$

and RHL of f(x) at x = 0

$\lim\limits_{x \to 0^+} f(x)] = \lim\limits_{x \to 0^+} \frac{x-|x|}{x} = \lim\limits_{h \to 0^+} \frac{(0+h)-|0+h|}{0+h}$

[putting x = 0 + h as x $\to$ 0, then b $\to$ 0]

$= \lim\limits_{h \to 0^+} \frac{h-h}{h} = 0$

Here, LHL $\neq$ RHL

$\therefore \lim\limits_{x \to 0} f(x)$ does not exist.

14. Here f (x) = 3 cot x + 5 cosec x

$\therefore$ f'$(x)=\frac{d}{dx}[3\cot x+5\text{cosec } x]$

$=3\frac{d}{dx}(\cot x)+5\frac{d}{dx}(cosecx)$

= - 3cosec^{2}x - 5 cosec x cot x

15. We have, f(x) = sinx + cosx

By using first principle of derivative

$f'(x)=\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$\therefore f'(x)=\lim\limits_{h\to 0}\frac{\sin(x+h)+\cos(x+h)-\sin x-\cos x}{h}$

= $\lim\limits_{h\to 0}\frac{[\sin x\cdot\cos h+\cos x\cdot\sin h+\cos x\cdot\cos h-\sin x\cdot\sin h-\sin x-\cos x]}{h}$ [$\because$ sin (x + y) = sin x cos y + cos x sin y and cos (x + y) = cos x cos y - sin x sin y]

$=\lim\limits_{h\to 0}\frac{[(\cos x\cdot\sin h-\sin x\cdot\sin h)+(\sin x\cdot\cos h-\sin x)+(cosx\cdot cosh-cosx)]}{h}$

$=\lim\limits_{h\to 0}\frac{\sin h(\cos x-\sin x)+\sin x(\cos h-1)+\cos x(\cos h-1)}{h}$

$=\lim\limits_{h\to 0}\frac{\sin h}{h}(\cos x-\sin x)+\lim\limits_{h\to 0}\frac{\sin x(\cos h-1)}{h}+\lim\limits_{h\to 0}\frac{\cos x(\cos h-1)}{h}$

$=1\cdot(\cos x-\sin x)+\lim\limits_{h\to 0}\sin x\left[\frac{-(1-\cos h)}{h}\right]$

$+\lim\limits_{h\to 0}\cos x\left[\frac{-(1-\cos h)}{h}\right]\left[\because\lim\limits_{x\to 0}\frac{\sin x}{x}=1\right]$

$=(\cos x-\sin x)-\sin x\cdot\lim\limits_{h\to 0}\left(\frac{1-\cos h}{h}\right)-\cos x\cdot\lim\limits_{h\to 0}\left(\frac{1-\cos h}{h}\right)$

$=(\cos x-\sin x)-\sin x\cdot\lim\limits_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h\times\frac{h}{4}}\times\frac{h}{4}-\cos x\cdot\lim\limits_{h\to 0}\frac{2\sin^2\frac{h}{2}}{h\times\frac{h}{4}}\times\frac{h}{4}$

$=(\cos x-\sin x)-\sin x\cdot 2\cdot\frac{1}{4}\lim\limits_{\frac{h}{2}\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times h-\cos x\cdot 2\cdot\frac{1}{4}\lim\limits_{\frac{h}{2}\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2 h$

$=(\cos x-\sin x)-\frac{1}{2}\cdot\sin x\cdot(1)\times 0-\cos x\cdot\frac{1}{2}\cdot(1)\times 0\left[\because\lim\limits_{x\to 0}\frac{\sin x}{x}=1\right]$

= (cos x - sin x) - 0 - 0

= cos x - sin x