CBSE Test Paper 02

CH-11 Conic Sections

1. The angle between the tangents drawn from the origin to the circle = $(x-7)^2+(y+1)^2=25$ is
   a. $\frac{\pi}{3}$
   b. $\frac{\pi}{2}$
   c. $\frac{\pi}{6}$
   d. $\frac{\pi}{8}$
2. The number of points on X-axis which are at a distance c units (c $<$3) from (2, 3) is
   a. 2
   b. 0
   c. 1
   d. 3
3. The equations x = a cos θ , y = b sin θ, 0 ≤ θ <2π , a $\neq$ b, represent
   a. a parabola
   b. an ellipse
   c. a hyperbola
   d. a circle
4. $x=\frac{e^t+e^{-t}}{2}, y=\frac{e^t-e^{-t}}{2}; t \in R$ represents
   a. a circle
   b. an ellipse
   c. a parabola
   d. a hyperbola
5. Two perpendicular tangents to the circle $x^2+y^2=r^2$ meet at P. The locus of P is
   a. x + y = r
   b. $x^2+y^2=4r^2$
   c. $x^2+y^2=\frac{r^2}{2}$
   d. $x^2+y^2=2r^2$
6. Fill in the blanks:

   The equation of the ellipse having foci (0, 1), (0, -1) and minor axis of length 1 is ________.

7. Fill in the blanks:

   The equation of the parabola whose focus is the point (2, 3) and directrix is the line x - 4y + 3 = 0 is ________.

8. Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).

9. Find the centre and radius of each of the following circle. $x^2 + (y + 2)^2 = 9$

10. Find the equation of the hyperbola whose directrix is 2x + y -1, focus (1, 2) and eccentricity $\sqrt{3}$.

11. Find the equation of a circle of radius 5 units, passing through the origin and having its centre on the Y-axis.

12. Find the centre and radius of the circle. $x^2 + y^2 + 6x - 4y + 4 = 0$

13. Find the equation of the hyperbola, the length of whose latus rectum is 8 and eccentricity is $\frac{3}{\sqrt{5}}$.

14. Find the equation of the circle passing through the points (4, 1) and (6, 5) and whose centre is on the line 4x + y = 16.

15. Find the axes, eccentricity, latus-rectum and the coordinates of the foci of the hyperbola

    $25x^2 - 36y^2 = 225$.

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Solution

1. (b) $\frac{\pi}{2}$ **Explanation:** The angle between the Radius and tangent is 90^0
2. (b) 0

   Explanation: the shortest distance from x-axis to the point is 3.
3. (b) an ellipse

   Explanation: parametric form of ellipse.
4. (d) a hyperbola

   Explanation: $x = \frac{e^t + e^{-t}}{2}, y = \frac{e^t - e^{-t}}{2}; t \in R$ Squaring both sides of both the equation, we get $x^2 = \frac{(e^t + e^- t)^2}{4}$ and $y^2 = \frac{(e^t - e^- t)}{4}$ Subtracting one equation from another we get $x^2 - y^2 = 1$ which is nothing but equation of hyperbola
5. (d) $x^2 + y^2 = 2\ r^2$ **Explanation:** locus of P is a circle with centre at origin and radius $\sqrt{2r^2}$. This is known as the director circle of the circle $x^2 + y^2 = r^2$
6. $\frac{4x^2}{1} + \frac{4y^2}{5} = 1$
7. $16x^2 + y^2 + 8xy - 74x - 78y + 212 = 0$
8. Given, the vertex is (0, 0) and focus is at (0, 2) which lies on Y-axis.

   The Y axis is the axis of parabola.

   Therefore, equation of parabola is of the form

   $x^2 = 4ay$

   $x^2 = 4(2)y$ i.e., $x^2 = 8y$

9. Given equation of circle is $x^2 + (y + 2)^2 = 9$

   $\Rightarrow (x - 0)^2 + \{y - (- 2)\}^2 = 3^2$

   On comparing with $(x - h)^2 + (y - k)^2 = r^2$, we get

   h = 0, k = - 2 and r = 3

   Hence, centre of circle = (0, - 2) and radius = 3.

10. Let S (1, 2) be the focus and P (x, y) be a point on the hyperbola. Draw PM perpendicular from P on the directrix. Then,

SP = ePM [By definition]

$\Rightarrow \sqrt{(x-1)^2+(y-2)^2} = \sqrt{3}\left|\frac{2x+y-1}{\sqrt{2^2+1^2}}\right|$

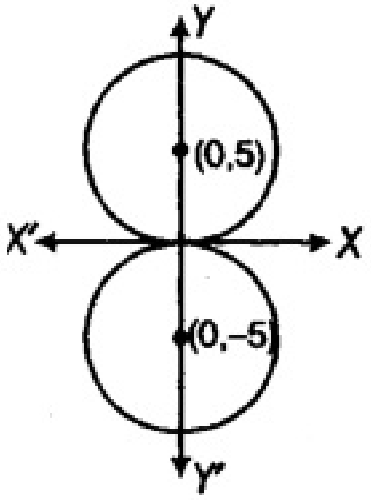
$\Rightarrow (x - 1)^2 + (y - 2)^2 = \frac{3(2x+y-1)^2}{5}$

$\Rightarrow 5[(x - 1)^2 +(y - 2)^2] = 3 (2 x + y - 1)^2$

$\Rightarrow 5x^2 + 5y^2 -10x - 20y + 25 = 3 (4x^2 + y^2 + 1 + 4xy - 4x - 2y)$

$\Rightarrow 7x^2 - 2y^2 + 12xy - 2x + 14y - 22 = 0$, which is the required equation of the hyperbola.

11. Radius of circle =5. The circle passes through the origin and its centre is on the Y-axis i.e., (0, 5) and (0, - 5). Equations of circle are



$(x - 0)^2 + (y - 5)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 10y + 25 = 25$

$\Rightarrow x^2 + y^2 - 10y = 0$

and $(x - 0)^2 + (y + 5)^2 = 5^2$

$\Rightarrow x^2 + y^2 +10y + 25 = 25$

$\Rightarrow x^2 + y^2 +10y = 0$

12. Given equation of circle is $x^2 + y^2 + 6x - 4y + 4 = 0$

$\Rightarrow (x^2 + 6x) + (y^2 - 4y) = - 4$

$\Rightarrow (x^2 + 6x + 9 - 9) + (y^2 - 4y + 4 - 4) = - 4$

$\Rightarrow (x^2 + 6x + 9) + (y^2 - 4y + 4) = - 4 + 4 + 9$

$\Rightarrow (x + 3)^2 + (y - 2)^2 = 9$

$\Rightarrow \{x - (-3)\}^2 + \{y - 2\}^2 = 3^2$

On comparing with $(x - h)^2 + (y - k)^2 = r^2$, we get

$h = -3$, $k = 2$ and $r = 3$

Hence, centre of circle = (- 3, 2) and radius = 3.

13. Let equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Now $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$

But $b^2 = a^2(e^2 - 1)$

$\therefore a^2(e^2 - 1) = 4a \Rightarrow a(e^2 - 1) = 4 \Rightarrow a\left(\frac{a}{5} - 1\right) = 4 \Rightarrow a = 5$

$\therefore b^2 = 4 \times 5 = 20$

Thus equation of required hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$

14. The equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$ .... (i)

Since the circle passes through point (4, 1)

$\therefore (4 - h)^2 + (1 - k)^2 = r^2 \Rightarrow 16 + h^2 - 8h + 1 + k^2 - 2k = r^2$

$\Rightarrow h^2 + k^2 - 8h - 2k + 17 = r^2$ .... (ii)

Also the circle passes through point (6, 5)

$\therefore (6 - h)^2 + (5 - k)^2 = r^2 \Rightarrow 36 + h^2 - 12h + 25 + k^2 - 10k = r^2$

$\Rightarrow h^2 + k^2 - 12h - 10k + 61 = r^2$

From (ii) and (iii), we have

$h^2 + k^2 - 8h - 2k + 17 = h^2 + k^2 - 12h - 10k + 61$

$\Rightarrow 4h + 8k = 44 \Rightarrow h + 2k = 11$ .... (iv)

Since the centre (h, k) of the circle lies on the line $4x + y = 16$

$\therefore 4h + k = 16$ . . . (v)

Solving (iv) and (v), we have

$h = 3$ and $k = 4$

Putting value of h and k in (ii), we have

$(3)^2 + (4)^2 - 8 \times 3 - 2 \times 4 + 17 = r^2$

$r^2 = 10$

Thus equation of required circle is

$(x - 3)^2 + (y - 4)^2 = 10 \Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y = 10$

$\Rightarrow x^2 + y^2 - 6x - 8y + 15 = 0$

15. We have,

$25x^2 - 36y^2 = 225$

$\Rightarrow \frac{25x^2}{225} - \frac{36y^2}{225} = 1$

$\Rightarrow \frac{x^2}{9} - \frac{4y^2}{25} = 1$

$\Rightarrow \frac{x^2}{9} - \frac{y^2}{\frac{25}{4}} = 1$

$\Rightarrow \frac{x^2}{(3)^2} - \frac{y^2}{\left(\frac{5}{2}\right)^2} = 1$

This is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a = 3 and b = $\frac{5}{2}$

The length of the transverse axis = 2a

$= 2 \times 3 = 6$

The length of the conjugate axis is $2b = 2 \times \frac{5}{2} = 5$

The eccentricity e is given by,

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$= \sqrt{1 + \frac{\frac{25}{4}}{9}}$

$= \sqrt{1 + \frac{25}{36}}$

$= \sqrt{\frac{61}{36}}$

$= \frac{\sqrt{61}}{6}$

Length of Latusrectum = $\frac{2b^2}{a} = \frac{25}{6}$

and Foci $\left(\pm\frac{\sqrt{61}}{2}, 0\right)$