CBSE Test Paper 01
CH-11 Conic Sections

1. The equation of the tangent to the conic $x^2 - y^2 - 8x + 2y + 11 = 0$ at (2, 1) is
   a. 2 x + 1 = 0
   b. x – 2 = 0
   c. x +2 = 0
   d. x + y + 1 = 0
2. The equation$(x^2 + y^2) + 5x - 7y - 2 = 0$ represents
   a. a circle
   b. an empty set
   c. a degenerate circle
   d. a pair of straight lines
3. Three normals to the parabola $y^2 = x$ are drawn through a point (c, 0) then
   a. none of these
   b. $c > \frac{1}{2}$
   c. $c = \frac{1}{2}$
   d. $c = \frac{1}{4}$
4. The graph of the function f(x) = $\frac{1}{x} i.e.$ the curve $y = \frac{1}{x}$ is
   a. a hyperbola
   b. a parabola
   c. an ellipse

d. a circle

5. The ellipse = $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, b^2=a^2$ is a

   a. a hyperbola

   b. none of these.

   c. horizontal ellipse

   d. vertical ellipse

6. Fill in the blanks:

   The equation of the circle having centre at (3, -4) and touching the line 5x + 12y - 12 = 0 is ________.

7. Fill in the blanks:

   ________ of the hyperbola is the ratio of the distance of any one focus from the centre and the distance of any one vertex from the centre.

8. Find the equation of parabola when the vertex is at (0, 0) and focus is at (0, 4).

9. What is the condition that the equation, on comparing with general equation of circle, $ax^2 + by^2 + 6x + 3y + hxy + 3 = 0$ is the equation of circle?

10. Find the equation of hyperbola having Foci (0, ±13) and the conjugate axis is of length 24.

11. Determine whether $x^2 + y^2 + 2x + 10y + 26 = 0$ represent a circle or point.

12. Find the equation of ellipse having Major axis on the x-axis and passes through the points (4, 3) and (6, 2)

13. Find the equation of ellipse having Length of minor axis 16, foci $(0,\pm6)$

14. Find the centre and radius of the circle. $x^2 + y^2 - 8x - 10y - 12 = 0$

15. Find the equation of the hyperbola whose foci are (4, 2) and (8,2) and eccentricity is 2.

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Solution

1. (b) x – 2 = 0

   Explanation: Differentiating the given equation w.r.t x, we get, 2x -2y$\frac{dy}{dx}$ -8 + 2 $\frac{dy}{dx}$ = 0 $\frac{dy}{dx}$(1-y) = x - 4 Therefore $\frac{dy}{dx}$ = $\frac{x-4}{1-y}$ Therefore $\frac{dy}{dx}$ (2,1) is not defined The equation of the tangent at (x_1, y_1) is y-y_1 = m(x - x1) Therefore the equation of the tangent is x-2 = 0
2. (a) a circle

   Explanation: The general equation of the circle is x^2+y^2-2gh-2fy+c = 0. Sice the given equations satisfies the general equation, it represents the equation of the circle.
3. (b) $c > \frac{1}{2}$ **Explanation:** The equation of the normal to a parabola y^2 = 4ax is y = mx -2am - am^3 Hence the equation of the normal to the given parabola y^2 = x is mx - $\frac{m}{2}$ - $\frac{m^3}{4}$ Since it passes throught (c,0) mc -$\frac{m}{2}$ - $\frac{m^3}{4}$ = 0 on solving we get m = 0 or m^2 = 4(c-1/2) If m = 0 then the equation of the normal is y = 0 If $m^2 \geq 0$, then 4(c-1/2) ≥ 0 Hence c-1/2 ≥ 0 or c > 1/2
4. (a) a hyperbola

   Explanation: it is called rectangular hyperbola.
5. (b) none of these.

   Explanation: If a^2 = b^2, then the equation becomes x^2 + y^2 = a^2 which represents the equation of a circle.
6. $(x - 3)^2 + (y + 4)^2 = \left(\frac{45}{13}\right)^2$
7. Eccentricity
8. Since, the vertex is at (0, 0) and focus is at (0, 4) which lies on Y-axis. The Y-axis is the axis of the parabola.

   $\therefore$ Equation of parabola is of the form

   $x^2 = -4ay \Rightarrow x^2 = -4(4)y$ [$\because$ a = 4]

   $\Rightarrow x^2 = -16y$
9. Given, equation will represent a circle, if Coefficient of x^2 = Coefficient of y^2

i.e., a = b and coefficient of xy should be zero.

i.e., h = 0.

10. Here foci are (0, ±13) which lie on y-axis.

So the equation of hyperbola in standard form is $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

$\therefore (13)^2 = a^2 + (12)^2 \Rightarrow a^2 = 169 - 144 = 25$

Thus required equation of hyperbola is

$\frac{y^2}{25}-\frac{x^2}{(12)^2}=1 \Rightarrow \frac{y^2}{25}-\frac{x^2}{144}=1$

11. We have, $x^2 + y^2 + 2x + 10y + 26 = 0$

On adding 1 and 25 both sides to make perfect squares, we get

$(x^2 + 2x + 1) + (y^2 + 10y + 25) = -26 + 1 + 25$

$\Rightarrow (x + 1)^2 + (y + 5)^2 =$

$\Rightarrow [x - (-1)]^2 + [y - (-5)]^2 = 0^2$

Hence, it represents a point circle, because it has zero radius.

12. Since the major axis is along x-axis.

So the equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Since the ellipse passes through point (4, 3)

$\therefore \frac{16}{a^2}+\frac{9}{b^2}=1\ldots$ (i)

Also the ellipse passes through point (6, 2)

$\therefore \frac{36}{a^2}+\frac{4}{b^2}=1$....(ii)

Solving (i) and (ii), we have

$a^2 = 52$ and $b^2 = 13$

Thus equation of required ellipse is

$\frac{x^2}{52}+\frac{y^2}{13}=1$

13. The foci $(0, \pm 6)$ lie on y-axis.

So the equation of ellipse in standard form is $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1]$

Now length of minor axis $2b = 16 \Rightarrow b = 8$

foci $(0, \pm c)$ is $(0, \pm 6) \Rightarrow c = 6$

We know that $c^2 = a^2 - b^2$

$\therefore (6)^2 = a^2 - (8)^2 \Rightarrow a^2 = 36 + 64 = 100$

Thus equation of required ellipse is

$\frac{x^2}{64}+\frac{y^2}{100}=1$

14. The given equation of circle is

$x^2 + y^2 - 8x - 10y - 12 = 0$

$\therefore (x^2 - 8x) + (y^2 + 10y) = 12$

Completing the square

$\Rightarrow [x^2 - 8x + (4)^2] + [y^2 + 10y + (5)^2]$

$= 12 + (4)^2 + (5)^2$

$\Rightarrow (x - 4)^2 + (y + 5)^2 = 12 + 16 + 25$

$\Rightarrow (x - 4)^2 + (y + 5)^2 = 53$

$\Rightarrow (x - 4)^2 + (y + 5)^2 = (\sqrt{53})^2$

Comparing it with $(x - h)^2 + (y - k)^2 = r^2$, we have

$h = 4$, $k = -5$ and $r = \sqrt{53}$

Thus coordinates of the centre is (4, -5) and radius is $\sqrt{53}$.

15. The centre of the hyperbola is the mid-point of the line joining the two foci.

So, the coordinates of the centre are $\left(\frac{4+8}{2}, \frac{2+2}{2}\right)$ i.e., (6, 2).

Let 2a and 2b be the length of transverse and conjugate axes and let e be the eccentricity.

Then, the equation of the hyperbola is

$\frac{(x-6)^2}{a^2}-\frac{(y-2)^2}{b^2} = 1$ ...(i)

Now, the distance between two foci = 2ae

$\Rightarrow \sqrt{(8-4)^2+(2-2)^2} = 2ae$ [$\because$ foci = (4, 2) and (8, 2)]

$\Rightarrow \sqrt{(4)^2} = 2ae$

$\Rightarrow 2ae = 4$

$\Rightarrow 2 \times a \times 2 = 4$ [$\because e = 2$]

$\Rightarrow a = \frac{4}{4} = 1$

$\Rightarrow a^2 = 1$

Now,

$b^2 = a^2 (e^2 - 1)$

$\Rightarrow b^2 = 1\ (2^2 - 1)$ [$\because e = 2$]

$\Rightarrow b^2 = 4 - 1$

$\Rightarrow b^2 = 3$

Putting $a^2 = 1$ and $b^2 = 3$ in equation (i), we get

$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$

$\Rightarrow \frac{3(x-6)^2-(y-2)^2}{3} = 1$

$\Rightarrow 3\ (x - 6)^2 - (y - 2)^2 = 3$

$\Rightarrow 3[x^2 + 36 - 12x] - [y^2 + 4 - 4y] = 3$

$\Rightarrow 3x^2 + 108 - 36x - y^2 - 4 + 4y = 3$

$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$

This is the equation of the required hyperbola.