CBSE Test Paper 02

CH-10 Straight Lines

1. If (x , y) are the coordinates of point in the plane , then $\begin{vmatrix} 3 & 4 & 2 \\ 5 & 8 & 2 \\ x & y & 2 \end{vmatrix} = 0$ represents
   a. a straight line parallel to x axis
   b. a straight line
   c. a circle
   d. none of these
2. A line L passes through the points (1 , 1) and (2 , 0) and another line M which is perpendicular to L passes through the point (1/2 , 0) . The area of the triangle formed by these lines with y axis is :
   a. 25/8
   b. 25/16
   c. none of these
   d. 25/4
3. The line which passes through the point (0 , 1) and perpendicular to the line x – 2y + 11 = 0 is
   a. none of these
   b. 2x + y – 1 = 0
   c. 2x – y + 1 = 0
   d. 2x – y + 3 = 0
4. The equation $y - y_1 = m(x - x_1), m \in R$, represents all lines through the point

(x_1, y_1) except the line

a. parallel to Y axis

b. parallel to the line x – y = 0

c. none of these

d. parallel to X axis

5. Two opposite vertices of a rectangle are (1 , 3) , (5 , 1) . If the equation of a diagonal of this rectangle is y = 2x + c , then the value of c is

a. 2

b. - 4

c. -9

d. 1

6. Fill in the blanks:

If a line is parallel to y-axis at a distance 'b' from y-axis then its equation is ________.

7. Fill in the blanks:

If a line is at a distance 'a' and parallel to x-axis, then the equation of the line is ________.

8. Find the slope of line, whose inclination is 30^{o}.

9. Find the new coordinates of point (3, -5), if the origin is shifted to the point (-3, - 2).

10. If two poles standing at the points A(1, - 2) and B (-3, 5), then find the distance between the poles.

11. Find the equations of the line which have slope $\frac{1}{2}$ and cuts off an intercept

i. -5 on Y-axis and
ii. 4 on X-axis.

12. Find the equation of the line joining the point (3,5) to the point of intersection of the lines $4x + y - 1 = 0$ and $7x - 3y - 35 = 0$.

13. Find the direction in which a straight line must be drawn through the point (-1, 2) so that its point of intersection with the line $x + y = 4$ may be at a distance of 3 units from this point.

14. Find the area of the triangle formed by the lines $y - x = 0$, $x + y = 0$ and $x - k = 0$.

15. In the $\triangle ABC$ with vertices A(2, 3), B(4, -1) and C(1, 2), find the equation and length of altitude from the vertex A.

Solution

1. (b) a straight line

   Explanation:

   The given determinant can be written as $2\begin{vmatrix} 3 & 4 & 1 \\ 5 & 8 & 1 \\ x & y & 1 \end{vmatrix}$ = 0

   On expansion we get

   2[3(8-y) - 4(5 - x) +1(5y - 8x)]

   On simplifying the equation 2(- 4x+2y+4) = 0 represents a striaght line.

2. (b) 25/16

   Explanation:

   The equation of the line joining the two points (x_1,y_1) and (x_2,y_2) is

   $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

   The given points are (1,1,) and (2,0)

   On substituting the values we get

   $\frac{y-1}{0-1} = \frac{x-1}{2-1}$

   On simplifying we get,

   x+y-2=0

   The line which is perpendicular to this line is x-y+k=0

   Since it passes through (1/2,0)

   (1/2) -0=k

This implies k= -1/2

Hence the equation of this line is x-y-1/2 = 0

On solving these twolines we get the point of intersection as (5/4,3/4)

The point which line x+y-2=0 cuts the Y axis is (0,2) and the point which the line x-y-1/2=0 cuts the Y axis is (0,-1/2)

Hend e the area of the triangle = [1/2]x[5/4]x[5/4] = 25/16 squnits

3. (b) 2x + y – 1 = 0
   Explanation: The line which is perpendicular to the given line is 2x + y + k = 0

   Since it passes through (0,1)

   2(0) + 1 + k = 0

   This implies k = -1

   Hence the equation of the required line is 2x + y - 1 = 0

4. (a) parallel to Y axis
   Explanation:

   The vertical lines which are parallel to Y axis has undefined slopes. Hence the slope of the line 'm' will be undefined.

   Therefore the above equation of the line will represent all lines through (x_1, y_1) except the line parallel to Y- axis

5. (b) - 4
   Explanation: Slope of the line joining the given points (1,3) and (5,1) is $\frac{1-3}{5-1}$ = -1/2

   Hence the line having slope -1/2 is y = [-1/2]x+c

   If the diagonal is y = 2x+c, then c = -4

6. x = $\pm$b

7. y = $\pm$a

8. Let θ be the inclination of a line with X-axis, then its slope = tan θ

   θ = 30^o

   $\Rightarrow$ Slope = tan 30^o
   $= \frac{1}{\sqrt{3}}$

9. The coordinates of the new origin are h = -3 and k = -2 and the original coordinates are given to be x = 3, y = -5.
   Let new coordinates of the point be (X, Y). Then,
   X = (x - h) = 3 - (-3)
   = 6 and
   Y = y - k = -5 - (-2) = -3
   Hence, the coordinates of the point (3, -5) in the new system are (6, -3).

10. Given points are A(1, - 2) and B (-3, 5).
    Now, distance between two poles
    $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
    $= \sqrt{(-3-1)^2 + (5+2)^2} = \sqrt{(-4)^2 + (7)^2}$
    $= \sqrt{16+49} = \sqrt{65}$

11. Given, m = slope of the line = $\frac{1}{2}$
    and c = intercept of the line on Y-axis = -5
    Hence, required equation of the line is
    $y = \frac{1}{2}x - 5 \Rightarrow$ x - 2y - 10=0
    Also, d = intercept of the line on X-axis = 4.
    Hence, required equation of the line is,
    $y = \frac{1}{2}(x-4) \Rightarrow$ x - 2y - 4=0

12. Let P(x,y) be the point of intersection of the lines 4x + y − 1 = 0 and 7x − 3y − 35 = 0.
    Now, y = 1 − 4x
    Thus, 7x − 3 (1 − 4x) − 35 = 0 [putting the value of y]
    7x − 3 + 12x − 35 = 0
    19x − 38

x = 2

$\Rightarrow$ y = 1 - 4 x - 1 - 8 = -7

$\therefore$ Let P (2, -7) and Q (3, 5)

The equation of the line PQ is

$y - y_1 = m(x - x_1)$

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - (-7) = \frac{5-(-7)}{3-2}(x - 2)$

y + 7 = 12 (x - 2)

y - 12x = -31

12x - y - 31 = 0

13. Let the required line makes an angle θ with the positive direction of x-axis. Then equation of line is

$\frac{x-(-1)}{\cos\theta} = \frac{y-2}{\sin\theta} = r \Rightarrow \frac{x+1}{\cos\theta} = \frac{y-2}{\sin\theta} = r$

It is given that r = 3

$\therefore \frac{x+1}{\cos\theta} = \frac{y-2}{\sin\theta} = 3$

$\therefore x + 1 = 3\cos\theta \Rightarrow x = 3\cos\theta - 1$

and $y - 2 = 3\sin\theta \Rightarrow y = 3\sin\theta + 2$

Since this point on the line x + y = 4

$\therefore 3\cos\theta - 1 + 3\sin\theta + 2 = 4$

$\therefore 3\cos\theta + 3\sin\theta = 3 \Rightarrow \cos\theta + \sin\theta = 1$

Squaring both sides, we have

$\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 1$

$\Rightarrow 1 + \sin 2\theta = 1 \Rightarrow \sin 2\theta = 0 \Rightarrow 2\theta = 0 \Rightarrow \theta = 0$

Which shows that required line is parallel to x-axis .

14. The equation of lines are

y - x = 0 .....(i)

x + y = 0.....(ii)

x -k = 0 ....(iii)

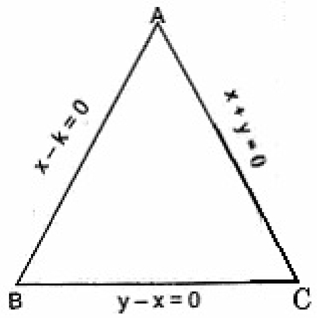
By solving (i) and (ii), we get the coordinates of point C.

$\therefore$ Coordinate of C are (0, 0).

By solving (ii) and (iii), we get the coordinates of point A.

$\therefore$ Coordinate of A are (k, -k).

By solving (i) and (iii), we get the coordinates of point B.



$\therefore$ coordinates of B are (k, k)

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} k & -k & 1 \\ k & k & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[(k^2 + k^2 + (0-0) + (0-0)\right]$

$= \frac{1}{2} \times 2k^2$

= k^2 sq. unit

15. Given, vertices of a $\triangle$ABC are A(2, 3), B(4, -1) and C(1, 2)

We know that altitude from a vertex of a triangle is perpendicular to the opposite side.

$\therefore$ Line AD $\perp$ line BC.

Then, the slope of AD $\times$ Slope of BC = -1

$\Rightarrow \quad m \times \frac{y_2 - y_1}{x_2 - x_1} = -1$

$\Rightarrow \quad m \times \frac{2+1}{1-4} = -1$ [$\because x_1 = 4, y_1 = -1, x_2 = 1, y_2 = 2$]

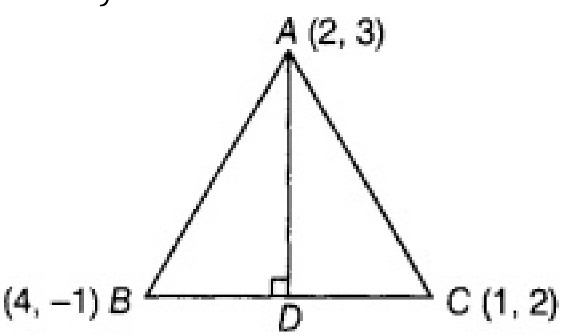
$\Rightarrow \quad m \times \frac{3}{-3} = -1 \Rightarrow m = 1$

Hence, equation of AD, by using $y - y_1 = m(x - x_1)$ is

y - 3 = 1(x - 2)

$\Rightarrow$ x - y - 2 + 3 = 0

$\Rightarrow$ x - y + 1 = 0



Now, equation of BC by using $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

is $y + 1 = \frac{2+1}{1-4}(x - 4)$ [∵ $x_1 = 4$, $y_1 = -1$, $x_2 = 1$, $y_2 = 2$]

$\Rightarrow \quad y + 1 = \frac{3}{-3}(x - 4)$

$\Rightarrow$ y + 1 = -x + 4

$\Rightarrow$ x + y - 3 = 0

Now, length of AD = Perpendicular distance from (2, 3) to the line BC

$= \left|\frac{2+3-3}{\sqrt{1^2+1^2}}\right| = \frac{2}{\sqrt{2}} = \sqrt{2}$ unit