CBSE Test Paper 02

Chapter 9 Some Applications of Trigonometry

1. If the angle of depression of an object from a 75m high tower is 30°, then the distance of the object from the tower is **(1)**
   a. $25\sqrt{3}$ m
   b. $50\sqrt{3}$ m
   c. $100\sqrt{3}$ m
   d. $75\sqrt{3}$ m
2. Two men are on opposite sides of a tower. They observe the angles of elevation of the top of the tower as 30° and 45° respectively. If the height of the tower is 100m, then the distance between them is **(1)**
   a. $100(\sqrt{3}-1)m$
   b. $100(\sqrt{3}+1)m$
   c. $100(1-\sqrt{3})m$
   d. none of these
3. The __________ is the angle between the horizontal and the line of sight to an object when the object is below the horizontal level. **(1)**
   a. angle of projection
   b. angle of elevation
   c. None of these
   d. angle of depression
4. The ratio between the height and the length of the shadow of a pole is $\sqrt{3}$: 1, then the sun's altitude is **(1)**
   a. 45°
   b. 30°
   c. 75°
   d. 60°

5. The angle of elevation of the top of a tower from a point on the ground and at a distance of 30m from its foot is 30°. The height of the tower is **(1)**

   a. $30\sqrt{3}$ m
   b. 10 m
   c. $10\sqrt{3}$m
   d. 30 m

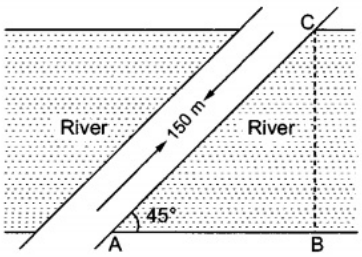
6. The angle of elevation of the top of a tower from a point 20 metres away from the base is 45°. Find the height of the tower. **(1)**

7. Find the angle of elevation of the sun (sun's altitude) when the length of the shadow of a vertical pole is equal to its height. **(1)**

8. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30^{o}. **(1)**

9. Form the top of a tower 50 m high the angles of depression of the top and bottom of a pole are observed to be 45^{o} and 60^{o} respectively. Find the height of the pole. **(1)**

10. A bridge across a river makes an angle of 45^{o} with the river bank as shown in Fig. If the length of the bridge across the river is 150 m, what is the width of the river? **(1)**



11. If two towers of height h_1 and h_2 subtends angles of 60° and 30° respectively at the mid points of line joining their feet, find $h_1 : h_2$ **(2)**

12. Find the length of kite string flying at 100 m above the ground with the elevation of 60^{o}. **(2)**

13. A window in a building is at height of 10 m from the ground. The angle of depression

of a point P on the ground from the window is 30^o .The angle of elevation of the top of the building from the point P is 60^o . Find the height of the building. **(2)**

14. From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30^o and 45^o respectively.
    Find:
    i. How far the pole is from the bottom of the tower,
    ii. the height of the pole.(Use $\sqrt{3}$ = 1.732) **(3)**

15. A path separates two walls. A ladder leaning against one wall rests at a point on the path. It reaches a height of 90 m on the wall and makes an angle of 60° with the ground. If while resting at the same point on the path, it were made to lean against the other wall, it would have made an angle of 45° with the ground. Find the height it would have reached on the second wall. **(3)**

16. If a hexagon ABCDEF circumscribe a circle, prove that AB + CD + EF = BC + DE + FA. **(3)**

17. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30^o. Another boy is standing on the roof of a 20 m high building and is flying his kite at an elevation of 45^o. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet. **(3)**

18. From the top of a hill, the angles of depression of two consecutive kilometre stones due east are found to be 45° and 30° respectively. Find the height of the hill. **(4)**

19. An aeroplane is flying at a height of 300 m above the ground. Flying at this height the angle of depression from the aeroplane of two points on both banks of a river are 45^o and 30^o respectively. Find the width of the river. **(4)**

20. The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of cloud in the lake is 60°. Find the height of the cloud. **(4)**

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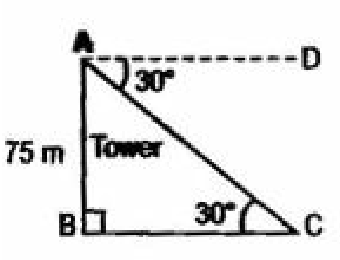
Solution

1. d. $75\sqrt{3}$ m

Explanation: In triangle ABC,



$\tan 30^\circ = \frac{AB}{BC}$

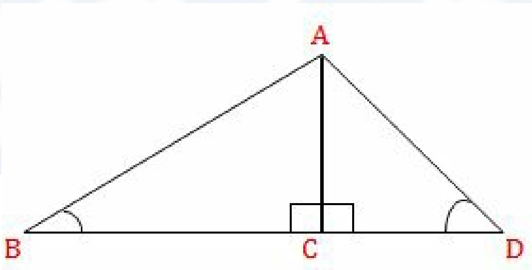
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC}$

$\Rightarrow$ BC = $75\sqrt{3}$ m

Therefore, the distance between P and foot of the tower is $75\sqrt{3}$ meters.

2. b. $100(\sqrt{3}+1)m$

Explanation:



Let the height of the tower AC = 100 m

Now, in triangle ABC,

$\tan 30^\circ = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BC}$

$\Rightarrow BC = 100\sqrt{3}$ m

Now, in triangle ACD,

$\tan 45^\circ = \frac{AB}{CD}$

$\Rightarrow 1 = \frac{100}{CD}$

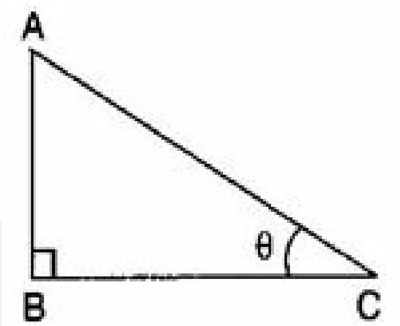
$\Rightarrow CD = 100$ m

Therefore, the required distance = BC + CD = $100\sqrt{3} + 100 = 100\left(\sqrt{3}+1\right)$ m

3. d. angle of depression

Explanation: The angle of depression is the angle between the horizontal and line of sight to an object when the object is below the horizontal level.
The angle of depression is formed when the observer is higher than the object he is looking at. It is the angle between the horizontal line and the line joining observer's eye and the object. It plays very important role in determining the heights and distances.

4. d. 60°

Explanation: 

Let the height of the pole be AB = $\sqrt{3}x$ meters and the length of the shadow be BC = x meters and angle of elevation = θ

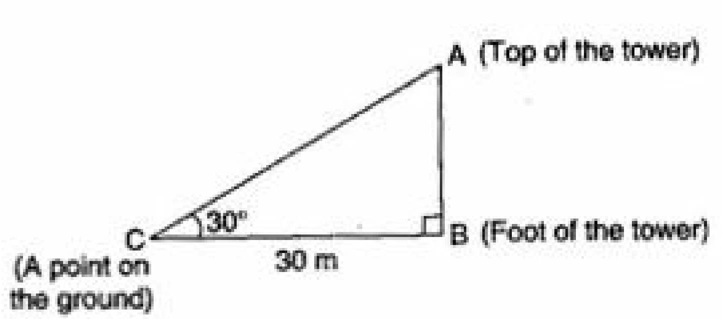
$\therefore \tan \theta = \frac{\sqrt{3}x}{x}$

$\Rightarrow \tan \theta = \sqrt{3}$

$\Rightarrow \tan \theta = \tan 60^{\circ}$

$\Rightarrow \theta = 60^{\circ}$

5. c. $10\sqrt{3}\,\mathrm{m}$

Explanation: 

In right triangle ABC,

$\tan 30^{\circ} = \frac{\mathrm{AB}}{\mathrm{BC}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\mathrm{AB}}{30}$

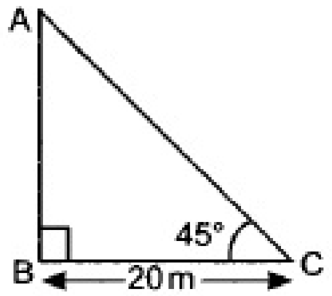
AB = $\frac{30}{\sqrt{3}}$ m

$\Rightarrow \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

= $10\sqrt{3}$ m

Hence the height of the tower is $10\sqrt{3}$ meters.

6. Let AB is the tower and C is the point 20 m away from the base of the tower

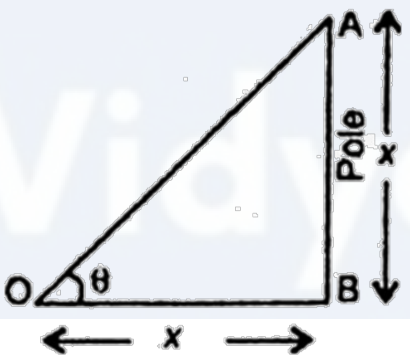


$\therefore$ BC = 20 m, $\angle$ACB = 45^{o}

In right $\triangle$ABC, tan 45° = $\frac{\text{AB}}{\text{BC}}$

$\Rightarrow$ 1 = $\frac{\text{AB}}{20}$

$\therefore AB = 20m$

7. 

According to the question,

Let height of pole (AB) = x m

Then, length of shadow (OB) = x m

In ΔOAB

$\tan\theta = \frac{AB}{OB}$

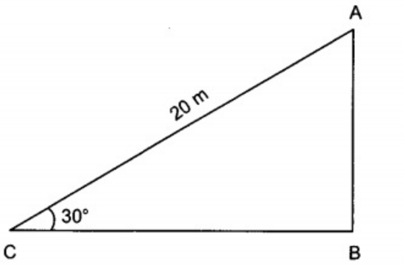
$\Rightarrow \tan\theta = \frac{x}{x}$

$\Rightarrow \tan\theta = 1 = \tan 45^{o}$

$\Rightarrow \theta = 45^{o}$

8. Let AB be the vertical pole and CA be the 20 m long rope such that its one end is tied from the top of the vertical pole AB and the other end C is tied to a point C on the

ground.



In $\triangle ABC$, we have

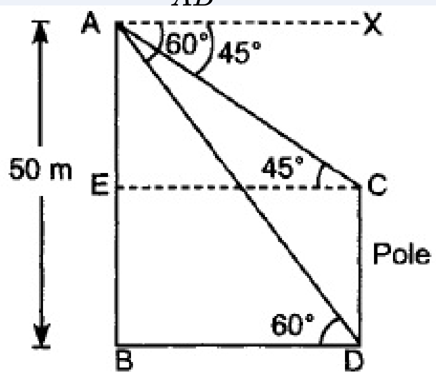
$\sin 30^\circ = \frac{AB}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{AB}{AC}$

$\Rightarrow$ AB = 10m.

Hence, the height of the pole is 10 m.

9. In $\triangle$ABD, $\frac{BD}{AB} = \cot 60^\circ$



$\Rightarrow \frac{\mathrm{BD}}{50} = \frac{1}{\sqrt{3}} \Rightarrow \mathrm{BD} = \frac{50}{\sqrt{3}}\mathrm{m}$

$\mathrm{BD} = \mathrm{EC} \Rightarrow \mathrm{EC} = \frac{50}{\sqrt{3}}\mathrm{m}$

In $\triangle AEC$, $\frac{\mathrm{AE}}{\mathrm{EC}} = \tan 45^\circ$

$\Rightarrow \quad \mathrm{AE} = \mathrm{EC} \Rightarrow \mathrm{AE} = \frac{50}{\sqrt{3}}\mathrm{m}$

Now $\mathrm{BE} = \mathrm{AB} - \mathrm{AE} = 50 - \frac{50}{\sqrt{3}}$

$= \frac{50(\sqrt{3}-1)}{\sqrt{3}}\mathrm{m}$

$DC = BE$= $\frac{50(\sqrt{3}-1)}{\sqrt{3}}\mathrm{m}$

10. Given that a bridge across a river makes an angle of 45^0 with the river bank as shown in Fig. If the length of the bridge across the river is 150 m, we have to find the width of the river.

In right triangle ABC, we have $\sin 45^\circ = \frac{BC}{AC}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{BC}{150}$

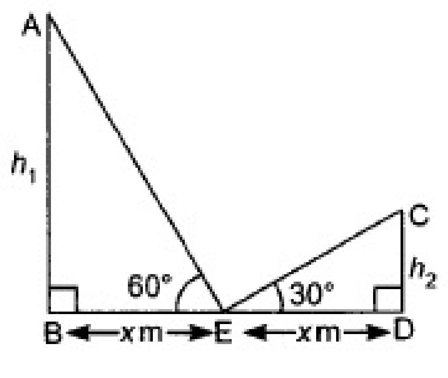
$\Rightarrow \quad BC = \frac{150}{\sqrt{2}}$(rationalization by √2)

$\Rightarrow \quad BC = 75\sqrt{2}\text{m}$

Hence, the width of the river is $75\sqrt{2}$ metres.

11. Let$AB\ and\ CD$ are towers of height $h_1\ and\ h_2$ respectively.

If E is the midpoint of BD then $BE = DE = x$



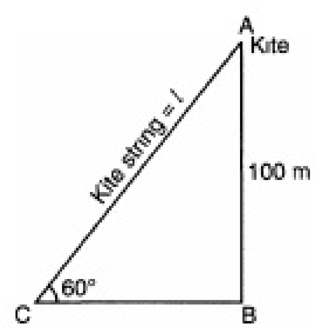
In right △ABE

$\frac{h_1}{x} = tan60^\circ$

$\Rightarrow$ h$_1$ = $\sqrt{3}x$

In right △CDE $\frac{h_2}{x}$ = tan 30° $\Rightarrow h_2 = \frac{x}{\sqrt{3}}$

Now $\frac{h_1}{h_2} = \frac{\sqrt{3}x}{\frac{x}{\sqrt{3}}} = \frac{3}{1} \Rightarrow h_1 : h_2 = 3 : 1$

12. 

Let the length of kite string AC = lm

$\angle ACB = 60^\circ$,.

height of kite AB = 100 m

From $\triangle ABC$, $\frac{AB}{BC} = \sin 60^\circ$(using Pythagoras theorem)

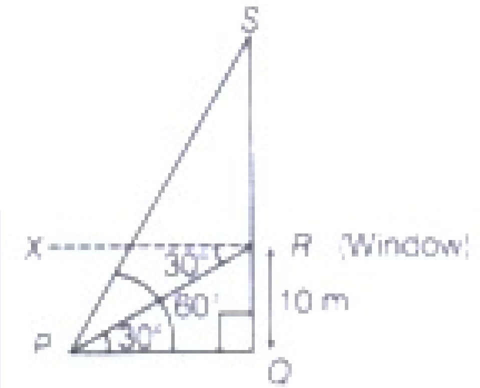
$\Rightarrow \quad \frac{100}{l} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad l = \frac{2\times100}{\sqrt{3}}$

$= \frac{200}{\sqrt{3}}\text{m}$

$= \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{200\sqrt{3}}{3} m$
Hence length the kite string = $\frac{200\sqrt{3}}{3}$

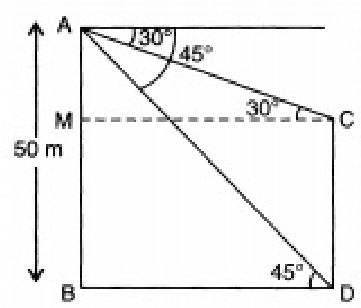
13. Let QS be the building and R be the position of the window in that figure.
    Given, height of the window is, $QR = 10m$
    $\angle QPR = \angle XRP = 30^o$ [alternate angles]
    and $\angle SPQ = 60^o$



In right-angled triangle ($\triangle PQR$), $\tan 30^o = \frac{P}{B} = \frac{QR}{PQ}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{PQ}$ [$\because \tan 30^o = \frac{1}{\sqrt{3}}$]
$\Rightarrow PQ = 10\sqrt{3}$...(i)
In right-angled triangle ($\triangle PQS$), $\tan 60^o = \frac{QS}{PQ}$
$\Rightarrow \sqrt{3} = \frac{QS}{10\sqrt{3}}$ [$\because \tan 60^o = \sqrt{3}$ and from Equation (i) $PQ = 10\sqrt{3}$]
$\Rightarrow QS = 10\sqrt{3} \times \sqrt{3}$
$\Rightarrow QS = 10 \times 3$
$\Rightarrow QS = 30\ m$
Therefore, The height of the building would be 30 meters.

14. Here, AB = 50 m, $\angle ADB = 45^\circ, \angle ACM = 30^\circ$
    $\tan 45^\circ = \frac{AB}{BD} = 1$
    $\Rightarrow$ AB = BD = 50 m.

    i. Therefore distance of pole from tower = BD = 50 meter
       Now in $\triangle AMC \tan 30^\circ = \frac{AM}{MC} = \frac{AM}{BD}$



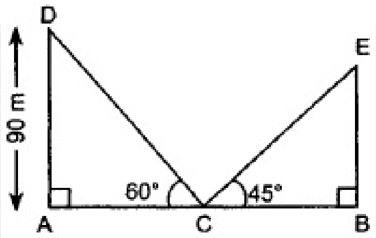
$$\frac{1}{\sqrt{3}} = \frac{AM}{BD} = \frac{AM}{50}$$

$$AM = \frac{50}{\sqrt{3}} = \frac{50}{1.73} = 28.90$$

ii. Hence height of pole = CD = MB

= AB - AM = 50 - 28.9 = 22.1 meter

15. Let AB is path



In rt. $\triangle$DAC, $\frac{DC}{AD} = \ cosec\, 60^\circ$

$\Rightarrow \frac{\text{DC}}{90} = \frac{2}{\sqrt{3}}$

$\text{DC} = \frac{2}{\sqrt{3}} \times 90\text{m} = \frac{180}{\sqrt{3}}\text{m}$

Now, $DC = CE$

$\therefore \quad \text{CE} = \frac{180}{\sqrt{3}}\text{m}$

In rt. $\triangle$EBC,

$\frac{BE}{CE} = \sin 45^\circ$

$\Rightarrow \quad \text{BE} = \frac{1}{\sqrt{2}} \times \frac{180}{\sqrt{3}}\text{m}$

$\Rightarrow \quad BE = 73.47\text{m}$

16. Hexagon ABCDEF touches a circle at G, H, I, J, K, L. So, from the external point tangents drawn on the circle are equal in length.

If A is external point and AG and AL are tangents, so

AG = AL ...(i)

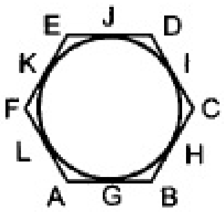
Similarly for B, BG = BH ...(ii)

Similarly for C, Cl = CH ... (iii)

Similarly for D, DI = DJ ... (iv)

EK = EJ ... (v)

and FK = FL ... (vi)



Adding (i), (ii), (iii), (iv), (v) and (vi), we get

AG + BG + CI + ID + EK + FK = BH + CH + DJ + EJ + FL + AL

$\Rightarrow$ (AG + BG) + (CI + ID) + (EK + FK) = (BH + CH) + (JD + EJ) + (FL + AL)

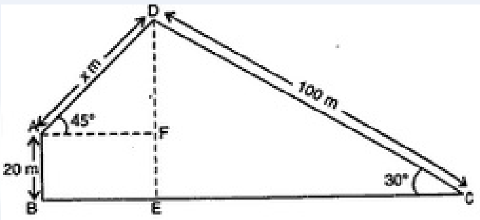
$\Rightarrow$ AB + CD + EF = BC + DE + FA.

17. Let C be the position of the first boy and AB be the building on the roof of which second boy is standing. Let the required length of the string be x m.

In, $\triangle ADF$ we have, sin $45^\circ = \frac{DF}{x}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{DF}{x} \Rightarrow DF = \frac{x}{\sqrt{2}}$

In $\triangle DEC$, we have sin $30^\circ = \frac{DE}{100}$



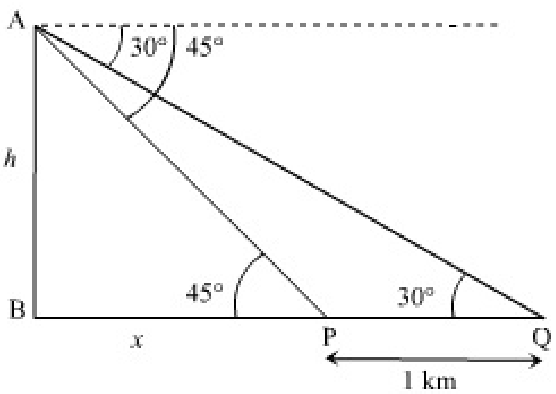
$\Rightarrow \frac{1}{2} = \frac{DF+20}{100}$ $[\because DE = DF + EF]$

$\Rightarrow DF = 50 - 20 = 30$

$\therefore 30 = \frac{\text{x}}{\sqrt{2}}$ From (i) we have

$\Rightarrow x = 30\sqrt{2} = 30 \times 1.41 = 42.32\text{m}$

$\therefore$ to meet the kites the second boy must have 42.32 m long string.

18. 

Given, AB is the hill and P and Q are two consecutive km stones.

Let the height of the hill AB be h m and

$BP = x\ m.$

PQ = 1km = 1000m

In ΔABP,

$\tan 45^\circ = \frac{AB}{BP}$

$\therefore 1 = \frac{h}{x}$

$\Rightarrow x = h$ ...(i)

In $\triangle$ABQ,

$\tan 30^\circ = \frac{\text{AB}}{\text{BQ}}$

$\therefore \frac{1}{\sqrt{3}} = \frac{h\text{m}}{(x+1000)\text{m}}$ $[\because \text{BQ} = \text{BP} + \text{PQ} = x + 1000]$

$\Rightarrow x + 1000 = \sqrt{3}h$

$\Rightarrow \sqrt{3}h = h + 1000$[Using (i)]

$\Rightarrow (\sqrt{3} - 1)h = 1000$

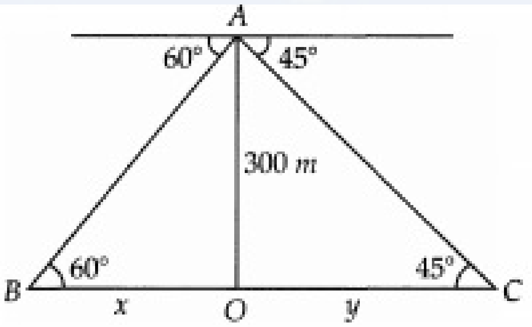
$\Rightarrow h = \frac{1000}{\sqrt{3}-1}$

$\Rightarrow h = \frac{1000(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{1000(\sqrt{3}+1)}{(3-1)}$

$= \frac{1000(\sqrt{3}+1)}{2}$

$= 500(\sqrt{3} + 1)$

19. 

Let height of the aeroplane AO = 300 m and BO be x and OC be y.

In $\triangle$AOC $\frac{OA}{OC} = \frac{300}{y} = tan 45^\circ = 1$

y = 300 m

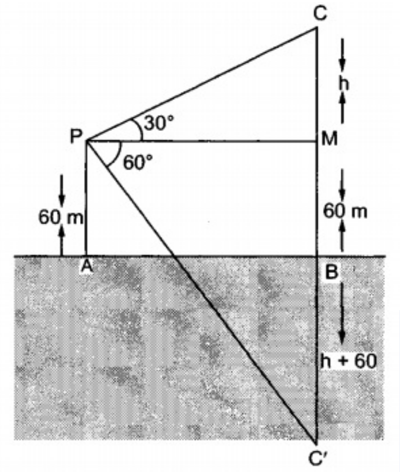
In $\triangle$ABO $\frac{OA}{OB} = \frac{300}{x} = tan 60^\circ = \sqrt{3}$

$x = \frac{300}{\sqrt{3}} = 100\sqrt{3} = 173.4$

So width of river

BC = x + y = 173.4 + 300 = 473.4 meter

20. Let AB be the surface of the lake and P be the point of observation such that AP = 60 metres. Let C be the position of the cloud and C be its reflection in the lake. Then, CB = C'B. Let PM be perpendicular from P on CB. Then,$\angle \mathrm{CPM} = 30^\circ$ and $\angle C'PM = 60^\circ$
Let CM = h. Then, CB = h + 60. Consequently, C'B = h + 60.
In $\triangle \mathrm{CMP},$ we have



$\tan 30^\circ = \frac{CM}{PM}$
$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{PM}$
or, PM = √3 h........(I)
In $\triangle PMC,'$ we have
$\tan 60^\circ = \frac{C'M}{PM}$
$\Rightarrow \quad \tan 60^\circ = \frac{CB+BM}{PM}$
$\Rightarrow \quad \sqrt{3} = \frac{h+60+60}{PM}$
$\Rightarrow \quad PM = \frac{h+120}{\sqrt{3}}$ ....(ii)
from equations (i) and (ii), we get
$\sqrt{3}h = \frac{h+120}{\sqrt{3}} \Rightarrow 3h = h + 120 \Rightarrow 2h = 120 \Rightarrow h = 60$
Now, CB = CM + MB = h+ 60 = 60 + 60 = 120.
Hence, the height of the cloud from the surface of the lake is 120 metres.