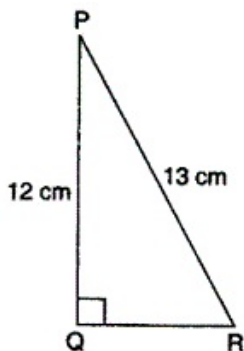


CBSE Test Paper 02
Chapter 8 Introduction to Trigonometry

1. The value of $\frac{\tan 30^\circ}{\cot 60^\circ}$ is **(1)**
 - a. 1
 - b. $\frac{1}{\sqrt{3}}$
 - c. $\frac{1}{\sqrt{2}}$
 - d. $\sqrt{2}$
2. $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$ **(1)**
 - a. $\sin 45^\circ$
 - b. 0
 - c. $\cos 45^\circ$
 - d. $\tan 45^\circ$
3. The value of $\tan 15^\circ \tan 20^\circ \tan 70^\circ \tan 75^\circ$ is **(1)**
 - a. 2
 - b. 0
 - c. 1
 - d. -1
4. If $x \cos A = 1$ and $\tan A = y$, then the value of $x^2 - y^2$ is **(1)**
 - a. -1
 - b. 0
 - c. 1
 - d. 2
5. If $\sin \theta = \frac{1}{2}$ and $\cos \phi = \frac{1}{2}$, then the value of $(\theta + \phi)$ is **(1)**
 - a. 0°
 - b. 30°
 - c. 90°
 - d. 60°
6. In a rectangle ABCD, $AB = 20\text{cm}$, $\angle BAC = 60^\circ$, calculate side BC and diagonals AC and BD. **(1)**
7. Convert the given trigonometric equation in the simplest form.

$$\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \quad (1)$$

8. Without using trigonometric tables, prove that: $(\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ) = 0$. (1)
9. Evaluate $(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ)$ (1)
10. If $\cos \theta = \frac{7}{25}$, write the value of $(\tan \theta + \cot \theta)$. (1)
11. If $\theta = 30^\circ$, verify that $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$. (2)
12. If $A = 60^\circ$ and $B = 30^\circ$, verify that $\cos (A - B) = \cos A \cos B + \sin A \sin B$. (2)
13. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$ (2)
14. If $\operatorname{cosec} A = 2$, find the value of $\frac{1}{\tan A} + \frac{\sin A}{1 + \cos A}$ (3)
15. If $a \cos \theta - b \sin \theta = x$ and $a \sin \theta + b \cos \theta = y$, prove that $a^2 + b^2 = x^2 + y^2$. (3)
16. Find the value of x if $4 \left(\frac{\sec^2 59^\circ - \cot^2 31^\circ}{3} \right) - \frac{2}{3} \sin 90^\circ + 3 \tan^2 56^\circ \times \tan^2 34^\circ = \frac{x}{3}$ (3)
17. If A, B, C, are the interior angles of a $\triangle ABC$, show that $\sin \frac{B+C}{2} = \cos \frac{A}{2}$. (3)
18. If $\sqrt{3} \cot^2 \theta - 4 \cot \theta + \sqrt{3} = 0$, then find the value of $\cot^2 \theta + \tan^2 \theta$. (4)
19. Find the value of other trigonometric ratios, given that $\tan \theta = \frac{2mn}{m^2 - n^2}$ (4)
20. In figure, find $\tan P - \cot R$. (4)



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Solution

1. a. 1

Explanation: Given: $\frac{\tan 30^\circ}{\cot 60^\circ}$
 $= \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = 1$

2. b. 0

Explanation: Given: $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ}$
 $= \frac{1 - (1)^2}{1 + (1)^2} = \frac{1 - 1}{1 + 1} = 0/2$
 $= 0$

3. c. 1

Explanation: Given: $\tan 15^\circ \tan 20^\circ \tan 70^\circ \tan 75^\circ$
 $= \tan 15^\circ \tan 20^\circ \tan (90^\circ - 20^\circ) \tan (90^\circ - 15^\circ)$
 $= \tan 15^\circ \tan 20^\circ \cot 20^\circ \cot 15^\circ$
 $= (\tan 15^\circ \cot 15^\circ) (\tan 20^\circ \cot 20^\circ)$
 $= 1 \times 1 = 1$

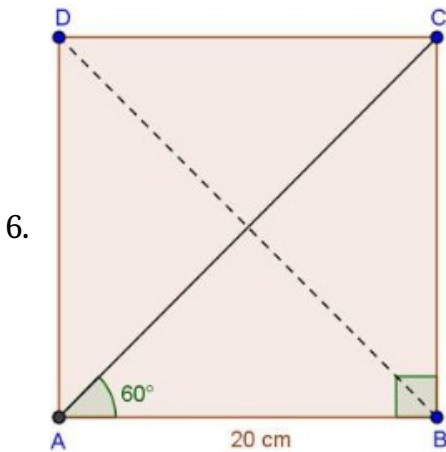
4. c. 1

Explanation: Given: $x \cos A = 1 \Rightarrow x = \frac{1}{\cos A} = \sec A$
And $\tan A = y$
 $\therefore x^2 - y^2 = \sec^2 A - \tan^2 A = 1$
[$\because \sec^2 \theta - \tan^2 \theta = 1$]

5. c. 90°

Explanation: Given: $\sin \theta = \frac{1}{2}$
 $\Rightarrow \sin \theta = \sin 30^\circ$
 $\Rightarrow \theta = 30^\circ$
And $\cos \phi = \frac{1}{2}$
 $\Rightarrow \cos \phi = \cos 60^\circ$
 $\Rightarrow \theta = 60^\circ$

$$\theta + \phi = 30^\circ + 60^\circ = 90^\circ$$



Since ABCD is a rectangle

then $\angle ABC = 90^\circ$

Now, in $\triangle ABC$

$$\tan(\angle BAC) = \frac{BC}{AB} \text{ and } \cos(\angle BAC) = \frac{AB}{AC}$$

$$\Rightarrow \tan 60^\circ = \frac{BC}{20} \text{ and } \cos 60^\circ = \frac{20}{AC}$$

$$\Rightarrow \sqrt{3} = \frac{BC}{20} \text{ and } \frac{1}{2} = \frac{20}{AC}$$

$$\Rightarrow BC = 20\sqrt{3} \text{ cm and } AC = 40 \text{ cm}$$

We know that in a rectangle, diagonals are equal

$$\therefore BD = AC = 40 \text{ cm}$$

7. We have,

$$\begin{aligned} & \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1 \end{aligned}$$

8. $(\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ) = 0$

$$\text{LHS} = (\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ)$$

$$= \sin^2(65^\circ) - \cos^2(25^\circ)$$

$$= \sin^2(90^\circ - 25^\circ) - \cos^2(25^\circ)$$

$$= \cos^2(25^\circ) - \cos^2(25^\circ)$$

$$= 0 = \text{RHS}$$

9. We know that, $\operatorname{cosec} 45^\circ = \sqrt{2}$, $\sec 30^\circ = (2/\sqrt{3})$, $\sin 30^\circ = (1/2)$, $\cot 45^\circ = 1$ & $\sec 60^\circ = 2$, putting these values in the given expression,

$$\text{we get:- } (\operatorname{cosec}^2 45^\circ \sec^2 30^\circ) (\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ)$$

$$\begin{aligned}
&= \left[(\sqrt{2})^2 \times \left(\frac{2}{\sqrt{3}} \right)^2 \right] \left[\left(\frac{1}{2} \right)^2 + 4 \times (1)^2 - (2)^2 \right] \\
&= \left[2 \times \frac{4}{3} \right] \left[\frac{1}{4} + 4 - 4 \right] \\
&= \frac{8}{3} \times \frac{1}{4} \\
&= \frac{2}{3}
\end{aligned}$$

10. Given, $\cos\theta = \frac{7}{25} \Rightarrow \cos^2\theta = \frac{49}{625}$
 $\Rightarrow \sin^2\theta = 1 - \cos^2\theta = 1 - \frac{49}{625} = \frac{576}{625}$
 $\Rightarrow \sin\theta = \frac{24}{25}$

Now, $\tan\theta + \cot\theta$

$$\begin{aligned}
&= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \\
&= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta} \\
&= \frac{1}{\sin\theta \cos\theta} \\
&= \frac{1}{\frac{24}{25} \times \frac{7}{25}} \\
&= \frac{625}{168}
\end{aligned}$$

11. According to the question, $\theta = 30^\circ$

L.H.S. = $\cos 3\theta$

Putting $\theta = 30^\circ$, we get

$$= \cos 3 \times 30^\circ = \cos 90^\circ = 0 \dots(i)$$

R.H.S. = $4\cos^3\theta - 3\cos\theta$

Putting $\theta = 30^\circ$, we get

$$\begin{aligned}
&= 4 \cos^3 30^\circ - 3 \cos 30^\circ \\
&= 4 \left(\frac{\sqrt{3}}{2} \right)^3 - 3 \times \frac{\sqrt{3}}{2} \\
&= \frac{4 \times 3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2} \\
&= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0 \dots(ii)
\end{aligned}$$

From (i) and (ii)

L.H.S. = R.H.S.

$$\Rightarrow \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

12. $A = 60^\circ$, $B = 30^\circ$

$$\text{LHS} = \cos (A - B) = \cos (60^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\text{RHS} = \cos A \cos B + \sin A \sin B$$

$$= \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{2\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{2}$$

LHS = RHS Hence verified.

$$13. \cos \theta + \sin \theta = \sqrt{2} \cos \theta$$

$$\Rightarrow \sin \theta = \sqrt{2} \cos \theta - \cos \theta$$

$$\Rightarrow \sin \theta = (\sqrt{2} - 1) \cos \theta$$

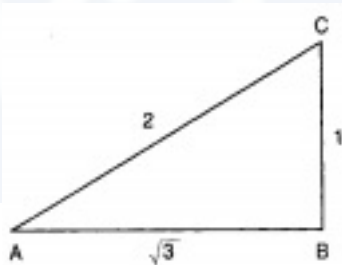
$$\Rightarrow \frac{\sin \theta}{\sqrt{2} - 1} = \cos \theta$$

$$\Rightarrow \frac{\sin \theta (\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \cos \theta$$

$$\Rightarrow \sqrt{2} \sin \theta + \sin \theta = \cos \theta$$

$$\Rightarrow \cos \theta - \sin \theta = \sqrt{2} \sin \theta = \text{RHS}$$

14.



According to the question,

$$\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{2}{1}$$

So, we draw a right triangle, right angled at B such that

Perpendicular = $BC = 1$, Hypotenuse = $AC = 2$

Using Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 2^2 = 1^2 + AB^2$$

$$\Rightarrow 4 - 1 = AB^2$$

$$\Rightarrow AB = \sqrt{3}$$

$$\tan A = \frac{BC}{AB} = \frac{1}{\sqrt{3}}, \sin A = \frac{BC}{AC} = \frac{1}{2} \text{ and } \cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \therefore \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} &= \frac{1}{\frac{\sqrt{3}}{1}} + \frac{1/2}{1+\frac{\sqrt{3}}{2}} \\ \Rightarrow \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} &= \frac{\sqrt{3}}{1} + \frac{1/2}{\frac{2+\sqrt{3}}{2}} = \frac{\sqrt{3}}{1} + \frac{1}{2+\sqrt{3}} \\ \Rightarrow \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} &= \sqrt{3} + \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} \\ \Rightarrow \frac{1}{\tan A} + \frac{\sin A}{1+\cos A} &= \sqrt{3} + \frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2} = \sqrt{3} + \frac{2-\sqrt{3}}{4-3} = \sqrt{3} + (2-\sqrt{3}) = 2 \end{aligned}$$

15. We have, $a \cos \theta - b \sin \theta = x \dots(i)$

and $a \sin \theta + b \cos \theta = y \dots(ii)$

Squaring Eq. (i) and (ii) and then adding, we get

$$x^2 + y^2 = (a \cos \theta - b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2$$

$$\Rightarrow x^2 + y^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$$

$$-2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta \quad [\because (a+b)^2 = a^2 + b^2 + 2ab$$

$$\text{and } (a-b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow x^2 + y^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta$$

$$\Rightarrow x^2 + y^2 = a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta) \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow x^2 + y^2 = a^2 + b^2$$

Hence proved, LHS = RHS

16. Given,

$$4 \left(\frac{\sec^2 59^\circ - \cot^2 31^\circ}{3} \right) - \frac{2}{3} \sin 90^\circ + 3 \tan^2 56^\circ \times \tan^2 34^\circ = \frac{x}{3}$$

$$\Rightarrow \frac{x}{3} = 4 \left(\frac{\operatorname{cosec}^2 31^\circ - \cot^2 31^\circ}{3} \right) - \frac{2}{3} + 3 \tan^2 (90^\circ - 34^\circ) \times \tan^2 34^\circ \quad [\text{Since, } \sec 59^\circ = \sec(90^\circ - 31^\circ) = \operatorname{cosec} 31^\circ]$$

$$\Rightarrow \frac{x}{3} = 4 \left(\frac{\operatorname{cosec}^2 31^\circ - \cot^2 31^\circ}{3} \right) - \frac{2}{3} + 3 \cot^2 34^\circ \times \tan^2 34^\circ \quad [\text{Since, } \tan(90^\circ - A) = \cot A]$$

$$\Rightarrow \frac{x}{3} = 4 \left(\frac{1}{3} \right) - \frac{2}{3} + 3 \quad [\text{Since, } \operatorname{cosec}^2 A - \cot^2 A = 1 \text{ \& } \tan A \cdot \cot A = 1]$$

$$\Rightarrow \frac{x}{3} = \frac{4}{3} - \frac{2}{3} + 3$$

$$\Rightarrow \frac{x}{3} = \frac{2}{3} + 3 = \frac{11}{3}$$

Hence, $x = 11$

17. In $\triangle ABC$, by angle sum property

$$A + B + C = 180^\circ$$

$$\Rightarrow B + C = 180^\circ - A \dots\dots(1)$$

Now, L.H.S

$$\begin{aligned}
& \sin \frac{B+C}{2} \\
&= \sin \left(\frac{180^\circ - A}{2} \right) \text{ [From (1)]} \\
&= \sin \left(\frac{180^\circ}{2} - \frac{A}{2} \right) \\
&= \sin \left(90^\circ - \frac{A}{2} \right) \text{ [sin(90}^\circ - \theta) = \cos \theta] \\
&= \cos \frac{A}{2} = RHS \text{ Hence Proved.}
\end{aligned}$$

18. Let $\cot \theta = x$,

$$\text{Then, } \sqrt{3} \cot^2 \theta - 4 \cot \theta + \sqrt{3} = 0$$

$$\text{or, } \sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$$

$$\sqrt{3}x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$(x - \sqrt{3})(\sqrt{3}x - 1) = 0$$

$$\therefore x = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$

$$\text{or, } \cot \theta = \sqrt{3} \text{ or } \cot \theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^\circ \text{ or } \theta = 60^\circ$$

When $\theta = 30^\circ$

$$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + \frac{1}{3} = \frac{10}{3}$$

When $\theta = 60^\circ$,

$$\cot^2 60^\circ + \tan^2 60^\circ$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2$$

$$= \frac{1}{3} + 3 = \frac{10}{3}$$

19. Given, $\tan \theta = \frac{2mn}{m^2 - n^2}$ (1)

We know that,

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$= 1 + \frac{(2mn)^2}{(m^2 - n^2)^2} \text{ [from (1)]}$$

$$= \frac{(m^2 - n^2)^2 + 4m^2n^2}{(m^2 - n^2)^2}$$

$$\Rightarrow \sec^2 \theta = \frac{(m^2 + n^2)^2}{(m^2 - n^2)^2}$$

$$\Rightarrow \sec \theta = \sqrt{\frac{(m^2+n^2)^2}{(m^2-n^2)^2}}$$

$$\Rightarrow \sec \theta = \frac{m^2+n^2}{m^2-n^2} \dots\dots(2)$$

Now,

$$\cos \theta = \frac{1}{\sec \theta} = \frac{m^2-n^2}{m^2+n^2} \text{ [from (2)]}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{m^2-n^2}{2mn} \text{ [from (1)]}$$

$$\text{Also, } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \tan \theta \cdot \cos \theta = \sin \theta$$

$$\Rightarrow \frac{2mn}{m^2-n^2} \times \frac{m^2-n^2}{m^2+n^2} = \sin \theta$$

$$\Rightarrow \sin \theta = \frac{2mn}{m^2+n^2}$$

$$\& , \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{m^2+n^2}{2mn}$$

20. In $\triangle PQR$, $\therefore \angle Q = 90^\circ$

$\therefore PR^2 = PQ^2 + QR^2$ By Pythagoras theorem

$$\Rightarrow (13)^2 = (12)^2 + QR^2$$

$$\Rightarrow 169 = 144 + QR^2$$

$$\Rightarrow QR^2 = 169 - 144 \Rightarrow QR^2 = 25$$

$$\Rightarrow QR = \sqrt{25} = 5\text{cm}$$

$$\therefore \tan P - \cot R = \frac{QR}{PQ} - \frac{QR}{PQ} = 0$$