1. Given that $\sin \alpha = \frac{1}{\sqrt{2}}$ and $\cos \beta = \frac{1}{\sqrt{2}}$, then the value of $(\alpha + \beta)$ is **(1)**

   a. $90°$

   b. $45°$

   c. $60°$

   d. $30°$

2. cot A tan A **(1)**

   a. tan A

   b. sec A

   c. 1

   d. cot A

3. If A and B are acute angles and sin A = cos B, then the value of (A + B) is **(1)**

   a. $0°$

   b. $90°$

   c. $30°$

   d. $60°$

4. Which of the following statement is true: **(1)**

   a. $\frac{\cos ec A}{\sin A} = \cos A$

   b. $\frac{\cos A}{\sin A} = \sec A$

   c. $\frac{\sin A}{\cos ec A} = \cot A$

   d. $\frac{\sin A}{\cos A} = \tan A$

5. Choose the correct option and justify your choice:sin 2A = 2 sin A is true when A = **(1)**

   a. 45°

   b. 0°

   c. 30°

   d. 60°

6. Express $\cos 83° - \sec 76°$ in terms of trigonometric ratios of angles between $0°$ and $45°$. **(1)**

7. Prove the trigonometric identity:

$\cos^4 A - \cos^2 A = \sin^4 A - \sin^2 A$ **(1)**

8. Prove that $\cos 1° \cos 2° \cos 3°...\cos 180° = 0$. **(1)**

9. If $\sin 5\theta = \cos 4\theta$, where 5θ and 4θ are acute angles, find the value of θ. **(1)**

10. If $\sec 5A = cosec(A + 30^o)$ where 5A is an acute angle, then find the value of A. **(1)**

11. Prove that sin (50° + θ) - cos (40° - θ) + tan 1° tan10° tan 20° tan70° tan 80° tan 89° =1. **(2)**

12. Prove the following identity : $\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A \cos^2 A$ **(2)**

13. Verify that, $\cos 60° = \cos^2 30° - \sin^2 30° = \frac{1}{2}$. **(2)**

14. If sin 3θ = cos (θ - 6°), where 3θ and θ - 6 ° are both acute angles, find the value of θ. **(3)**

15. Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$, using identity $sec^2\theta = 1 + tan^2\theta$. **(3)**

16. Prove the identity:

    $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$ **(3)**

17. Find the value of the following without using trigonometric tables:

    $\frac{\cos 50°}{2\sin 40°} + \frac{4(cosec^2 59° - \tan^2 31°)}{3\tan^2 45°} - \frac{2}{3}$ tan 12° tan 78°. sin 90° **(3)**

18. Evaluate : $\frac{\cos 65°}{\sin 25°} - \frac{\tan 20°}{\cot 70°}$ - sin 90° + tan 5° tan 35° tan 60° tan 55° tan 85°. **(4)**

19. If $\tan\theta = \frac{1}{\sqrt{5}}$

    i. Evaluate : $\frac{cosec^2\theta - sec^2\theta}{cosec^2\theta + sec^2\theta}$

    ii. Verify the identity : $\sin^2\theta + \cos^2\theta = 1$. **(4)**

20. Given the value of $\sec\theta = \frac{13}{12}$, Calculate all other trigonometric ratios. **(4)**

Solution

1. a. $90°$

   Explanation: Given: $\sin \alpha = \dfrac{1}{\sqrt{2}}$

   $\Rightarrow \sin \alpha = \sin 45°$

   $\Rightarrow \alpha = 45°$

   And $\cos \beta = \dfrac{1}{\sqrt{2}}$

   $\Rightarrow \cos \beta = \cos 45°$

   $\Rightarrow \beta = 45°$

   $\therefore \alpha + \beta = 45° + 45° = 90°$

2. c. 1

   Explanation: $\cot A \tan A = \cot A \times \dfrac{1}{\cot A} = 1 \times 1 = 1$

3. b. $90°$

   Explanation: Given: $\sin A = \cos B$

   Since A and B are acute angles, then

   $\sin A = \sin(90° - B)$

   $\Rightarrow A = 90° - B$

   $\Rightarrow A + B = 90°$

4. d. $\dfrac{\sin A}{\cos A} = \tan A$

   Explanation: In right Ld triangle ABC rt Ld at B

   $\text{Sin } A = \dfrac{BC}{AC}$

   $\text{Cos } A = \dfrac{AB}{AC}$

Then $\dfrac{SinA}{CosA} = \dfrac{\frac{BC}{AC}}{\frac{AB}{AC}}$

$\dfrac{BC}{AC} \times \dfrac{AC}{AB} = \dfrac{\frac{BC}{AC}}{\frac{AB}{AC}}$

$L.H.S = \dfrac{\sin A}{\cos A} = \dfrac{BC}{AB}$

$R.H.S = tanA$
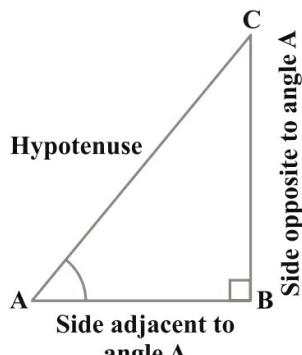
$= \dfrac{sinA}{cosA} = \dfrac{BC}{AB} = \dfrac{BC}{AB}$

L.H.S = R.H.S

Therefore $\dfrac{sinA}{cosA}$ = tan A is true.

5. b. 0°

   Explanation: When A = 0^0

   $\sin 2A = \sin 2\left(0^\circ\right) = 0$

   $2\sin A = 2\sin(0^\circ) = 2(0) = 0$

   $\therefore \sin 2A = 2\sin 2A$

   Hence, the correct option is (A) = 0^0

6. cos 83^0 - sec 76^0 = cos (90^0 - 7^0) - sec (90^0 - 14^0)

   = (sin 7^0 - cosec 14^0).

7. We have,

   LHS = $\cos^4 A - \cos^2 A$

   $\Rightarrow$ LHS = $\cos^2 A (\cos^2 A - 1)$

   $\Rightarrow$ LHS = $-\cos^2 A (1 - \cos^2 A)$

   $\Rightarrow$ LHS = $-\cos^2 A \sin^2 A = -(1 - \sin^2 A)\sin^2 A = -\sin^2 A + \sin^4 A$

   $\Rightarrow$ LHS = $\sin^4 A - \sin^2 A$ = RHS

8. We have,

   LHS = cos 1° cos2° cos3°...cos 180°

   = cos1° cos2° cos3°... cos89° cos90° cos 91°... cos180°

   = cos1°cos2°cos3°... $\times$ cos 89° $\times$ 0 $\times$ cos 91° $\times$ ...cos180° = 0 = RHS $\left[\because \cos 90^\circ = 0\right]$

9. We have,

   $\sin 5\theta = \cos 4\theta$

   $\Rightarrow \quad \sin 5\theta = \sin(90^\circ - 4\theta)\left[\because Sin(90^\circ - \theta) = Cos\theta\,\right]$

$\Rightarrow \quad 5\theta = 90° - 4\theta$

$\Rightarrow \quad 9\theta = 90°$

$\Rightarrow \quad \theta = 10°$

10. Given, $\sec 5A = cosec(A + 30^o)$

cosec(90⁰ - 5A) = cosec(A + 30⁰) [$\because cosec(90^o - \theta) = sec\theta$]

$90° - 5A = A + 30°$

$90° - 30° = A + 5A$

$\Rightarrow 6A = 60^o$

Therefore, $A = 10^o$

11. **LHS** = sin(50⁰ + θ) - cos (40⁰ - θ) + tan 1⁰ tan 10⁰ tan 20⁰ tan 70⁰ tan 80⁰ tan 89⁰

= [sin (90⁰ - (40 - θ))] - cos (40 - θ) + tan (90⁰ - 89⁰) tan (90⁰ - 80⁰) tan (90⁰ - 70⁰) × tan 70⁰ tan 80⁰ tan 89⁰

= cos (40⁰ - θ) - cos (40⁰ - θ) + cot 89⁰ × cot 80⁰ × cot 70⁰ × $\frac{1}{\cot 70°}$ × $\frac{1}{\cot 80°}$ × $\frac{1}{\cot 89°}$

= 0 + 1

= 1 = RHS

12. To prove : sin⁶ A + cos⁶ A = 1 - 3 sin² A cos² A

LHS = sin⁶ A + cos⁶ A

= (sin² A)³ + (cos² A)³

= (sin² A + cos² A)³ - 3sin² A . cos² A(sin² A + cos² A) [$\because$a³ + b³ = (a + b)³ - 3ab(a + b)]

= 1³ - 3sin² A . cos² A × 1

= 1 - 3sin² A . cos² A

= RHS.

Hence proved.

13. We have,

$\cos 30° = \frac{\sqrt{3}}{2}, \sin 30° = \frac{1}{2}$

therefore,

R.H.S = $\cos^2 30° - \sin^2 30°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

and, L.H.S = $\cos 60° = \frac{1}{2}$

$\therefore$ L.H.S = R.H.S

i.e, $\cos 60° = \cos^2 30° - \sin^2 30° = \frac{1}{2}$

14. Given,

$\sin 3\theta = \cos (\theta - 6°)$

$\cos (90° - 3\theta) = \cos (\theta - 6°)$

$90° - 3\theta = \theta - 6°$

$4\theta = 90° + 6° = 96°$

$\therefore \quad \theta = \frac{96°}{4} = 24°$

15. We have to prove that, $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ using identity $sec^2\theta = 1 + tan^2\theta$

LHS = $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta}$ [dividing the numerator and denominator by $\cos\theta$.]

$= \frac{(\tan\theta + \sec\theta) - 1}{(\tan\theta - \sec\theta) + 1} = \frac{\{(\tan\theta + \sec\theta) - 1\}(tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$ [Multiplying and dividing by $(\tan\theta - \sec\theta)$]

$= \frac{(\tan^2\theta - \sec^2\theta) - (tan\theta - \sec\theta)}{\{(\tan\theta - \sec\theta) + 1\}(\tan\theta - \sec\theta)}$ [$\because (a - b)(a + b) = a^2 - b^2$]

$= \frac{-1 - \tan\theta + \sec\theta}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)}$ [$\because \tan^2\theta - \sec^2\theta = -1$]

$= \frac{-(\tan\theta - \sec\theta + 1)}{(\tan\theta - \sec\theta + 1)(\tan\theta - \sec\theta)} = \frac{-1}{\tan\theta - \sec\theta}$

$= \frac{1}{\sec\theta - \tan\theta}$ = RHS

Hence Proved.

16. Take ,

LHS = 2($\sin^6\theta$ + $\cos^6\theta$) - 3 ($\sin^4\theta$ + $\cos^4\theta$) + 1

$\Rightarrow$ LHS = 2 $\{(\sin^2\theta)^3 + (\cos^2\theta)^3\}$ - 3 $\{(\sin^2\theta)^2 + (\cos^2\theta)^2\}$ + 1

Using $a^3 + b^3 = (a + b)^3 - 3ab (a + b)$ and $a^2 + b^2 = (a + b)^2 - 2ab$ in above expression, where a = $\sin^2\theta$ & b = $\cos^2\theta$; we get :-

LHS = 2 $\{(\sin^2\theta + \cos^2\theta)^3 - 3 \sin^2\theta \cos^2\theta (\sin^2\theta + \cos^2\theta)\}$ -3 $\{(\sin^2\theta + \cos^2\theta)^2 - 2 \sin^2\theta \cos^2\theta\}$ + 1

$\Rightarrow$ LHS = 2 (1 - 3$\sin^2\theta \cos^2\theta$) - 3 (1 - 2$\sin^2\theta \cos^2\theta$) + 1 [Since, $\sin^2 A + \cos^2 A = 1$]

$\Rightarrow$ LHS = 2 - 6 $\sin^2\theta \cos^2\theta$ - 3 + 6 $\sin^2\theta \cos^2\theta$ + 1

Hence, L.H.S. = 0 = R.H.S.

Hence, proved.

17. We know that,

cos50° = cos (90°-40)° = sin 40°

$cosec^2$ 59° = $cosec^2$ (90° - 31°) = sec^2 31°

and tan 78° = tan (90° -12°) = cot 12°

Now,

$$= \frac{\cos 50°}{2\sin 40°} + \frac{4\left(cosec^2 59° - \tan^2 31°\right)}{3\tan^2 45°} - \frac{2}{3}tan12°tan78°sin90°$$

$$= \frac{\sin 40°}{2\sin 40°} + \frac{4\left(\sec^2 31° - \tan^2 31°\right)}{3\times(1)^2} - \frac{2}{3}\tan 12° \cot 12° \times 1$$

$$= \frac{1}{2} + \frac{4}{3} - \frac{2}{3} = \frac{7}{6}$$

18. First, we need to solve given equation in parts

$$\Rightarrow \frac{\cos 65°}{\sin 25°} = \frac{\cos 65°}{\sin(90° - 65°)} = \frac{\cos 65°}{\cos 65°} = 1$$

$$\Rightarrow \frac{\tan 20°}{\cot 70°} = \frac{\tan(90° - 70°)}{\cot 70°} = \frac{\cot 70°}{\cot 70°} = 1$$

$$\Rightarrow \sin 90° = 1$$

$$\Rightarrow \tan 5°\tan 35°\tan 60°\tan 55°\tan 85°$$

= tan (90° - 85°) tan (90° - 55°) tan 55°tan 60°tan 85°

= cot 85°tan 85°cot 55°tan 55°.$\sqrt{3}$

$$= 1 \times 1 \times \sqrt{3} = \sqrt{3}$$

∴ Given Expression = 1 - 1 - 1+ $\sqrt{3}$

Therefore, $\sqrt{3}$ - 1 is the answer.

19. Given, $\tan\theta = \frac{P}{B} = \frac{AB}{BC} = \frac{1}{\sqrt{5}}$
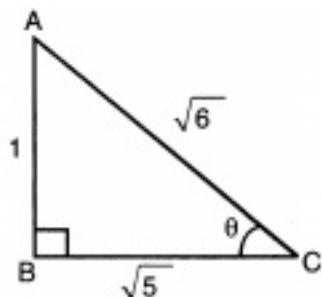
In △ABC, $AC^2 = AB^2 + BC^2 = 1 + 5 = 6$

∴ AC = $\sqrt{6}$

i. $\frac{cosec^2\theta - \sec^2\theta}{cosec^2\theta + \sec^2\theta}$

$$= \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$

$$= \frac{\cot^2\theta-\tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$

$$= \frac{(\sqrt{5})^2-\left(\frac{1}{\sqrt{5}}\right)^2}{2+(\sqrt{5})^2+\left(\frac{1}{\sqrt{5}}\right)^2}$$

$$= \frac{5-\frac{1}{5}}{2+5+\frac{1}{5}}$$

$$= \frac{25-1}{35+1}$$

$$= \frac{24}{36} = \frac{2}{3}$$

ii. LHS = $\sin^2\theta + \cos^2\theta$

$$= \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$

$$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1 = \text{R.H.S}$$

20. Let us draw a right triangle ABC in which $\angle BAC = \theta$

$\sec\theta = \frac{13}{12}, \ldots,$ Given

$\Rightarrow \frac{AC}{AB} = \frac{13}{12} \Rightarrow \frac{AC}{13} = \frac{AB}{12} = k(say)$

where k is a positive number.

$\Rightarrow AC = 13k, AB = 12k$
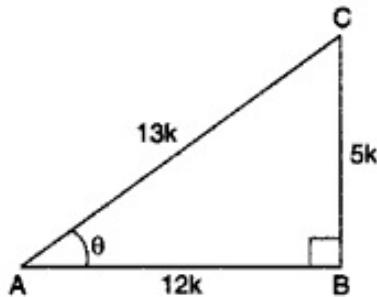
By using the Pythagoras theorem, we have

AC² = AB² + BC²

$\Rightarrow (13k)^2 = (12k)^2 + BC^2$

$\Rightarrow 169k^2 = 144k^2 + BC^2$

$\Rightarrow BC^2 = 169k^2 - 144k^2$

$\Rightarrow BC^2 = 25k^2 \Rightarrow BC = \sqrt{25}\,k^2$

$\Rightarrow BC = 5k$

Therefore, $\sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$$\left.\begin{aligned} \cos\theta &= \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13} \\ \cos\theta &= \frac{1}{\sec\theta} = \frac{12}{13} \end{aligned}\right\}$$

$$\left.\begin{aligned} \tan\theta &= \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12} \\ \tan\theta &= \frac{\sin\theta}{\cos\theta} = \frac{\frac{5}{13}}{\frac{13}{13}} = \frac{5}{12} \end{aligned}\right\}$$

$$\left.\begin{aligned} cosec\theta &= \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5} \\ cosec\theta &= \frac{1}{\sin\theta} = \frac{13}{5} \end{aligned}\right\}$$

$$\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5} \text{ or } \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{5}{12}} = \frac{12}{5}$$