CBSE Test Paper 01

Chapter 05 Arithmetic Progression

1. In an AP, if a = 4, n = 7 and a_n = 4, then the value of 'd' is **(1)**
   a. 0
   b. 1
   c. 3
   d. 2
2. The next two terms of the AP : k, 2k + 1, 3k + 2, 4k + 3, ………… are **(1)**
   a. 5k + 4 and 6k + 5
   b. 4k + 4 and 4k + 5
   c. 5k + 5 and 6k + 6
   d. 5k and 6k
3. The common difference of the A.P. can be **(1)**
   a. only negative
   b. only zero
   c. positive, negative or zero
   d. only positive
4. The 7th term from the end of the A.P. – 11, – 8, – 5, ……., 49 is **(1)**
   a. 28
   b. 31
   c. -11
   d. -8
5. The common difference of the A.P whose $a_n = -3n + 7$ is **(1)**
   a. 3
   b. 1
   c. -3
   d. 2

6. If 5 times the 5th term of an AP is equal to 10 times the 10th term, show that its 15th term is zero. **(1)**

7. Write the first term a and the common difference d of A.P. -1.1, - 3.1, -5.1, - 7.1,... **(1)**

8. For what value of n are the n^{th} term of the following two AP's are same 13, 19, 25, ... and 69, 68, 67 .... **(1)**

9. Find k, if the given value of x is the k^{th} term of the given AP $5\frac{1}{2}$, 11, $16\frac{1}{2}$, 22, ..., x = 550. **(1)**

10. Find the 6^{th} term from the end of the A.P. 17,14,11,..., - 40 **(1)**

11. How many terms of the AP 17,15,13,11,... must be added to get the sum 72? **(2)**

12. Find n. Given a = first term = -18.9, d = common difference = 2.5, a_n = the nth term = 3.6, n = ? **(2)**

13. Find the number of terms in each of the following APs. 18, $15\frac{1}{2}$, 13, ..., – 47. **(2)**

14. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find the common difference and the number of terms. **(3)**

15. The 14th term of an A.P. is twice its 8th term. If the 6th term is -8, then find the sum of its first 20 terms. **(3)**

16. Find the 6th term from end of the AP 17, 14, 11, ..., –40. **(3)**

17. The houses of a row in a colony are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x. **(3)**

18. The sum of first n terms of an A.P. is $3n^2 + 4n$. Find the 25^{th} term of this A.P. **(4)**

19. If sum of first 6 terms of an A.P. is 36 and that of the first 16 terms is 256, find the sum of the first 10 terms. **(4)**

20. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP. **(4)**

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Answers

1. a. 0

   Explanation: Given: a = 4, n = 7 and a_n = 4, then

   $a_n = a + (n-1)\,d$

   $\Rightarrow 4 = 4 + (7-1)\,d$

   $\Rightarrow$4 - 4 = 6d

   $\Rightarrow 6d = 0$

   $\Rightarrow d = 0$

2. a. 5k + 4 and 6k + 5

   Explanation: Given: $k, 2k+1, 3k+2, 4k+3, \ldots\ldots\ldots\ldots\ldots$

   Here $d = 2k + 1 - k = k + 1$

   Therefore, the next two terms are

   $4k + 3 + k + 1 = 5k + 4$ and $5k + 4 + k + 1 = 6k + 5$

3. c. positive, negative or zero

   Explanation: The common difference of the A.P. can be positive, e.g. 1, 2, 3, 4 ..... d is +ve and series is increasing negative e.g 4, 3, 2, 1 ...... d is - ve and series is decreasing

   or zero also and the AP becomes constant e.g 4, 4, 4, 4 ....

4. b. 31

   Explanation: Reversing the given A.P., we have

   $49, 46, 43, \ldots\ldots\ldots\ldots\ldots\ldots, -11$

   Here, $a = 49, d = 46 - 49 = -3$ and $n = 7$

   $\therefore a_n = a + (n-1)\,d$

   $\Rightarrow a_7 = 49 + (7-1) \times (-3)$

   = $49 + 6 \times (-3)$

   $\Rightarrow a_7 = 49 - 18 = 31$

5. c. – 3

   Explanation: Given: $a_n = -3n + 7$

Putting $n = 1, 2, 3$, we get

$a = -3 \times 1 + 7 = -3 + 7 = 4$

$a_2 = -3 \times 2 + 7 = -6 + 7 = 1$

$a_3 = -3 \times 3 + 7 = -9 + 7 = -2$

$\therefore$ Common difference $(d) = a_2 - a = 1 - 4 = -3$

6. Let 1[st] term = a and common difference = d.

   a_5 = a + 4d, a_{10} = a + 9d

   According to the question, 5 $\times$ a_5 = 10 $\times$ a_{10} $\Rightarrow$ 5(a + 4d) = 10(a + 9d) $\Rightarrow$ 5a + 20d = 10a + 90d $\Rightarrow$ a = -14d

   Now, a_{15} = a + 14d $\Rightarrow$ a_{15} = -14d + 14d = 0.

7. -1.1, - 3.1, -5.1, - 7.1,...

   First term (a) = -1.1

   We know that common difference is difference between any two consecutive terms of an A.P.

   So, common difference(d) = (-3.1) - (-1.1)

   = -3.1 + 1.1

   = -2

8. n^{th} term of 13, 19, 25, ... = nth term of 69, 68, 67, ...

   13 + (n - 1) 6 = 69 + (n - 1) (-1)

   13 + 6n - 6 = 69 - n + 1

   n + 6n = 70 - 7

   7n = 63

   n = 9

   Therefore, n = 9

9. a = $5\frac{1}{2} = \frac{11}{2}$, d = a_2 - a_1 = 11 - $\frac{11}{2}$ = $\frac{11}{2}$ and x = 550

   A.T.Q., a_k = x

   $\Rightarrow a + (k-1)d = 550$

   $\Rightarrow \frac{11}{2} + (k-1)\frac{11}{2}$ = 550

   $\Rightarrow \frac{11}{2} + \frac{11}{2}k - \frac{11}{2}$ = 550

   $\Rightarrow \frac{11}{2}k = 550$

$\Rightarrow$ k = $\frac{550\times 2}{11} = 100$

10. A.P. is 17,14,11,..., - 40

We have,

l = Last term = -40 , a = 17 and, d = Common difference= 14 - 17 = - 3

$\therefore$ 6th term from the end = l - (n -1)d

= l - (6-1) d

= -40 - 5 $\times$ (-3)

= -40 + 15

= -25

So, 6th term of given A.P. is -25.

11. Given A.P. is 17, 15, 13, 11........

Here, 1st term (a) = 17 and common difference (d) = (15 - 17) = -2

Let the sum of n terms be 72. Then,

S_n = 72

$\Rightarrow \frac{n}{2}\cdot$ {2a + (n - 1)d} = 72

$\Rightarrow$ n$\cdot$ {2 × 17 + (n - 1)(-2)} = 144

$\Rightarrow$ n(36 - 2n)=144

$\Rightarrow 2n^2$- 36n + 144 = 0

$\Rightarrow n^2$ - 18n + 72 = 0

$\Rightarrow n^2$ - 12n - 6n + 72 = 0

$\Rightarrow$ n(n - 12) - 6(n - 12) = 0

$\Rightarrow$(n - 12)(n - 6) = 0

$\Rightarrow$ n = 6 or n = 12.

$\therefore$ sum of first 6 terms = sum of first 12 terms = 72.

This means that the sum of all terms from 7th to 12th is zero.

12. a_n = a + (n - 1)d

$\Rightarrow$ 3.6 = - 18.9 + (n - 1) (2.5)

$\Rightarrow$ 3.6 + 18.9 = (n - 1) (2.5)

$\Rightarrow$ 22.5 = (n - 1) (2.5)

$\Rightarrow n - 1 = \frac{22.5}{2.5}$

$\Rightarrow$ n - 1 = 9

$\Rightarrow$ n = 10

13. $18, 15\frac{1}{2}, 13, \ldots, -47$

Here, a = 18

$d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = -\frac{5}{2}$

a_n = -47

Let the number of terms be n.

Then,

a_n = -47

$\Rightarrow$ a + (n - 1)d = -47

$\Rightarrow 18 + (n-1)\left(-\frac{5}{2}\right) = -47$

$\Rightarrow -\frac{5}{2}(n-1) = -47 - 18$

$\Rightarrow -\frac{5}{2}(n-1) = -65$

$\Rightarrow \frac{5}{2}(n-1) = 65$

$\Rightarrow n - 1 = \frac{65\times2}{5}$

$\Rightarrow$ n - 1 = 26

$\Rightarrow$ n = 26 + 1

$\Rightarrow$ n = 27

Hence, the number of terms of the given AP is 27.

14. Let the given AP contains n terms.

First term, a = 5

Last term, l = 45

S_n = 400

$\Rightarrow \frac{n}{2}$ [a + l] = 400

$\Rightarrow \frac{n}{2}$ [5 + 45] = 400

$\Rightarrow$n $\times$ 50 = 800

$\Rightarrow$n = 16

Thus, the given AP contains 16 terms.

Let d be the common difference of the given AP.

then,

T_{16} = 45

$\Rightarrow$a + 15d = 45

$\Rightarrow 5 + 15d = 45$

$\Rightarrow 15d = 40$

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$.

Therefore, common difference of the given AP is $\frac{8}{3}$.

15. Let first term be a and common difference be d.

Here, $a_{14} = 2a_8$

a + 13d = 2(a + 7d)

a + 13d = 2a + 14d

a = - d...(i)

$a_6 = -8$

a + 5d = -8 ... (ii)

Putting the value of a from (i) in (ii), we get

-d + 5d = -8

4d = -8

d = -2

Put d = -2 in (i)

a = -(-2)

a = 2

So ,a = 2, d = - 2

$S_{20} = \frac{20}{2}[2 \times 2 + (20 - 1)(-2)]$

$= 10[4 + 19 \times (-2)]$

= 10(4 - 38)

= 10 $\times$ (-34)

= - 340. Which is the required sum of first 20 terms.

16. The given AP is 17, 14, 11, ..., – 40

Here, a = 17

d = 14 - 17 = -3

l = -40

Let there be n terms between in the given AP

Then, nth term = -40

$\Rightarrow$ a + (n - 1)d = -40 $\because a_n = a + (n-1)d$

$\Rightarrow$ 17 + (n - 1) (-3) = -40

$\Rightarrow$ (n - 1) (-3) = -40 - 17

$\Rightarrow$ (n - 1) (-3) = -57

$\Rightarrow n-1=\frac{-57}{-3}$

$\Rightarrow$ n - 1= 19

$\Rightarrow$ n = 19 + 1

$\Rightarrow$ n = 20

Hence, there are 20 terms in the given AP.

Now, 6th term from the end

= (20 - 6 + 1)th term from the beginning

= 15th term from the beginning

= a + (15 - 1)d $\because a_n=a+(n-1)d$

= 17 + 14 (-3)

= 17 - 42

= - 25

Hence, the 6th term from the end of the given AP is -25.

17. According to the question, we have to find the value of x.

We are given an AP, namely 1,2,3,..., (x -1), x, (x +1),..., 49

such that 1 + 2 + 3 +... + (x -1) = (x +1) + (x + 2) +... + 49.

Thus, we have S_{x-1} = S_{49}- S_x ... (i)

Using the formula, $S_n=\frac{n}{2}$ (a + l) in (i), we have,

$\frac{(x-1)}{2}\cdot\{1+(x-1)\}=\frac{49}{2}\cdot(1+49)-\frac{x}{2}\cdot(1+x)$

$\Rightarrow\frac{x(x-1)}{2}+\frac{x(x+1)}{2}=1225$

$\Rightarrow 2x^2 = 2450 \Rightarrow x^2 = 1225 \Rightarrow x = \sqrt{1225} = 35$

Hence, x = 35.

18. According to the question,

Sum of n terms of the A.P. $S_n = 3n^2 + 4n$

$S_1 = 3 \times 1^2 + 4 \times 1 = 7 = t_1$ ....(i)

$S_2 = 3 \times 2^2 + 4 \times 2 = 20 = t_1 + t_2$ ....(i)

$S_3 = 3 \times 3^2 + 4 \times 3 = 39 = t_1 + t_2 + t_3$ .... (iii)

From (i), (ii), (iii)

$t_1 = 7$, $t_2 = 13$, $t_3 = 19$

Common difference, d = 13 - 7 = 6

25^{th} of the term of this A.P., $t_{25} = 7 + (25 - 1)6$

= 7 + 144 = 151

∴ The 25^{th} term of the A.P. is 151.

19. Consider the A.P. whose first term and common difference are ‘a’ and ‘d’ respectively.

If sum of first 6 terms of an A.P. is 36.

$S_6 = 36$

$\therefore \frac{6}{2}$ [2a + (6 – 1)d] = 36 $\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

⇒ 3[2a + 5d] = 36

⇒ 2a + 5d = $\frac{36}{3}$

⇒ 2a + 5d = 12 ...(i)

If sum of first 16 terms is 256,

So, $S_{16} = 256$

⇒ $\frac{16}{2}$ [2a + (16 – 1)d] = 256

⇒ 8[2a + 15d] = 256

⇒ 2a + 15d = $\frac{256}{8}$

⇒ 2a + 15d = 32 ...(ii)

Subtracting (i) from (ii), we get

$$\begin{array}{rl} 2a + 15d = 32 & \ldots\text{(ii)} \\ 2a + \ \ 5d = 12 & \text{[From (i)]} \\ - \quad - \qquad - & \\ \hline 10d = 20 & \end{array}$$

$$\begin{array}{r} 2a + 15d = 32 \\ 2a + \ \ 5d = 12 \\ - \quad - \qquad - \\ \hline 10d = 20 \end{array}$$

⇒ d = 2

Now, 2a + 5d = 12 [From (i)]

⇒ 2a + 5(2) = 12

⇒ 2a + 10 = 12

⇒ 2a = 12 – 10

⇒ a = $\frac{2}{2}$

⇒ a = 1

Hence, a = 1 and d = 2

So, S_{10} = $\frac{10}{2}$ [2a + (10 – 1)d]

= 5[2(1) + 9(2)]

= 5[2 + 18]

= 5[20]

= 100

⇒ S_{10} = 100

Hence, the sum of first 10 terms is 100.

20. Let the first term and the common difference of the AP be a and d respectively.

According to the question,

Third term + seventh term = 6

⇒ [a + (3 - 1)d] + [a + (7 - 1)d] = 6 = a + (n - 1)d

⇒ (a + 2d) + (a + 6d) = 6 ⇒ 2a + 8d = 6

⇒ a + 4d = 3 ..... (1)

Dividing throughout by 2 &

(third term) (seventh term) = 8

⇒ (a + 2d) (a + 6d) = 8

⇒ (a + 4d - 2d) (a + 4d + 2d) = 8

⇒ (3 - 2d) (3 + 2d) = 8

⇒ 9 - 4d^2 = 8

$\Rightarrow 4d^2 = 1 \Rightarrow d^2 = \frac{1}{4} \Rightarrow d + \pm\frac{1}{2}$

Case I, when $d = \frac{1}{2}$

Then from (1), $a + 4\left(\frac{1}{2}\right) = 3$

⇒ a + 2 = 3 ⇒ a = 3 - 2 ⇒ a = 1

∴ Sum of first sixteen terms of the AP = S_{16}

$= \frac{16}{2}[2a + (16 - 1)d] \because S_n = \frac{n}{2}[2a + (n - 1)d]$

= 8[2a + 15d]

$= 8[2(1) + 15(\frac{1}{2})]$

$= 8[12 + \frac{15}{2}]$

$= 8[\frac{19}{2}]$

$= 4 \times 19 = 76$

Case II. When $d = -\frac{1}{2}$

Then from (1),

$a + 4\left(-\frac{1}{2}\right) = 3$

$\Rightarrow$ a - 2 = 3 $\Rightarrow$ a = 3 + 2 $\Rightarrow$ a = 5

$\therefore$ Sum of first sixteen terms of the AP = S_{16}

$= \frac{16}{2}[2a + (16 - 1)d] \because S_n = \frac{n}{2}[2a + (n - 1)d]$

$= 8[2a + 15d] = 8\left[2(5) + 15\left(-\frac{1}{2}\right)\right] = 8\left[10 - \frac{15}{2}\right] = 8\left[\frac{5}{2}\right] = 20$