CBSE Test Paper 02

Chapter 4 Quadratic Equation

1. Rohan's mother is 26 years older than him. The product of their ages 3 years from now will be 360, then Rohan's present age is **(1)**
   a. 6 years
   b. 7 years
   c. 10 years
   d. 8 years
2. If $ax^2 + bx + c = 0$ has equal roots, then c is equal to **(1)**
   a. $\frac{b^2}{2a}$
   b. $\frac{b^2}{4a}$
   c. $\frac{-b^2}{4a}$
   d. $-\frac{b^2}{2a}$
3. Which of the following is a quadratic equation? **(1)**
   a. $x^3 - x^2 = (x-1)^3$
   b. $x^2 + 2x + 1 = (4-x)^2 + 3$
   c. $-2x^2 = (5-x)\left(2x - \frac{2}{5}\right)$
   d. $(k+1)x^2 + \frac{3}{2}x - 5 = 0$, where k = – 1
4. If 'sin α' and 'cos α' are the roots of the equation $ax^2 + bx + c = 0$, then b^2 = **(1)**
   a. $a^2 + 2ac$
   b. $a^2 + ac$
   c. $a^2 - ac$
   d. $a^2 - 2ac$
5. The quadratic equation, sum of whose roots is $3\sqrt{2}$ and their product is 5, is **(1)**
   a. $x^2 + 3\sqrt{2}x - 5 = 0$

b. $x^2+3\sqrt{2}x+5=0$
c. $x^2-3\sqrt{2}x-5=0$
d. $x^2-3\sqrt{2}x+5=0$

6. State whether the following equation is quadratic equation in x?
   $2x^2 + \frac{5}{2}x - \sqrt{3} = 0$ **(1)**

7. If -2 is a root of the equation $3x^2 + 5x + 2k = 0$, then find the value of k. **(1)**

8. Find two consecutive numbers whose squares have the sum 85. **(1)**

9. Check whether the given equation is quadratic equation: $(x+1)^2 = 2(x-3)$ **(1)**

10. Find discriminant of the quadratic equation: $5x^2 + 5x + 6 = 0$. **(1)**

11. The sum of the squares of two positive integers is 208. If the square of the larger number is 18 times the smaller number, find the numbers. **(2)**

12. Solve the quadratic equation by factorization: $3x^2-2\sqrt{6}x+2=0$ **(2)**

13. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speeds of the two trains. **(2)**

14. The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, then find the two numbers. **(3)**

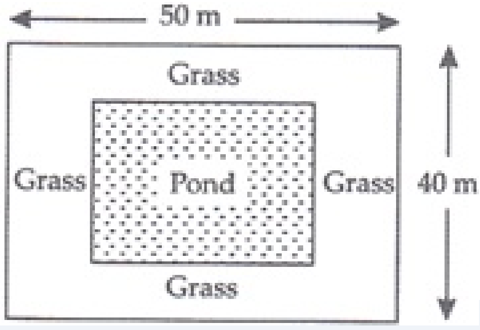
15. A swimming pool is filled with three pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately. **(3)**

16. Two numbers differ by 3 and their product is 504. Find the numbers. **(3)**

17. The sum of first 2 even natural numbers is given by the relation x = n (n + 1). Find n, if the sum is 420. **(3)**

18. The length of the hypotenuse of a right-angled triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle. **(4)**

19. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2. Find the length and breadth of the pond **(4)**



20. A man travels a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km an hour, the journey would have taken two hours less. Find the original speed of the train. **(4)**

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Answer

1. b. 7 years

   Explanation: Let Rohan's present age be x years.
   Then Rohan's mother age will be $(x+26)$ years.
   And after 3 years their ages will be $(x+3)$ and $(x+29)$ years. According to question,
   $(x+3)(x+29)=360$
   $\Rightarrow x^2+29x+3x+87=360$
   $\Rightarrow x^2+32x-273=0$
   $\Rightarrow x^2+39x-7x-273=0$
   $\Rightarrow x(x+39)-7(x+39)=0$
   $\Rightarrow (x-7)(x+39)=0$
   $\Rightarrow x-7=0$ and $x+39=0$
   $\Rightarrow x=7$ and $x=-39$ [$x=-39$ is not possible]
   Therefore, Rohan's present is 7 years.

2. b. $\frac{b^2}{4a}$

   Explanation: If $ax^2+bx+c=0$ has equal roots, then
   $b^2-4ac=0$
   $\Rightarrow 4ac=b^2$
   $\Rightarrow c=\frac{b^2}{4a}$

3. a. $x^3-x^2=(x-1)^3$

   Explanation: In equation $x^3-x^2=(x-1)^3$
   $\Rightarrow x^3-x^2=x^3-1-3x^2+3x$
   $\Rightarrow -x^2+3x^2-3x+1=0$
   $\Rightarrow 2x^2-3x+1=0$
   It is a quadratic equation as its degree is 2.

4. a. a^2+2ac

   Explanation: Given: $\alpha=\sin\alpha$ and $\beta=\cos\alpha$
   $\because \alpha+\beta=\frac{-b}{a}$

$\therefore \sin\alpha + \cos\alpha = \frac{-b}{a}$

$\Rightarrow (\sin\alpha + \cos\alpha)^2 = \frac{b^2}{a^2}$

$\Rightarrow 1 + 2\sin\alpha\cos\alpha = \frac{b^2}{a^2}$ .....(i)

And $\alpha\beta = \frac{c}{a}$

$\Rightarrow \sin\alpha\cos\alpha = \frac{c}{a}$

$\Rightarrow 2\sin\alpha\cos\alpha = \frac{2c}{a}$......(ii)

Subtracting eq. (ii) from eq. (i), we get

$1 = \frac{b^2}{a^2} - \frac{2c}{a}$

$\Rightarrow 1 = \frac{b^2 - 2ac}{a^2}$

$\Rightarrow a^2 = b^2 - 2ac$

$\Rightarrow b^2 = a^2 + 2ac$

5. d. $x^2 - 3\sqrt{2}x + 5 = 0$

Explanation: Given: Sum of roots $(\alpha + \beta) = 3\sqrt{2}$ and Product of roots $(\alpha\beta) = 5$

$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 3\sqrt{2}x + 5 = 0$

6. We have, $2x^2 + \frac{5}{2}x - \sqrt{3} = 0$

$\Rightarrow 4x^2 + 5x - 2\sqrt{3} = 0$

Clearly, it is in the form of $ax^2 + bx + c = 0$

$2x^2 + \frac{5}{2}x - \sqrt{3} = 0$ is a quadratic equation.

7. put x = -2 in $3x^2 + 5x + 2k = 0$

$3(-2)^2 + 5(-2) + 2k = 0$

$3(-2)^2 + 5(-2) + 2k = 0$

12 - 10 = -2k

2=-2k

$\Rightarrow$ k = -1

$\therefore$ k = -1

8. Let the two consecutive numbers be x and x + 1.

According to question,

$x^2 + (x + 1)^2 = 85$

$x^2 + x^2 + 1 + 2x = 85$
$2x^2 + 2x = 85 - 1$
$2x^2 + 2x = 84$
$2(x^2 + x) = 84$
$x^2 + x - 42 = 0$
$x^2 + 7x - 6x - 42 = 0$
$x(x+7) - 6(x+7) = 0$
$(x+7)(x-6) = 0$
Hence, numbers are 6 and 7 or -7 and -6

9. The given equation is $(x+1)^2 = 2(x-3)$

   $\Longrightarrow x^2 + 2x + 1 - 2x + 6 = 0$

   $\Longrightarrow x^2 + 7 = 0$

   $\Longrightarrow x^2 + 0.x + 7 = 0$

   Which is of the form $ax^2 + bx + c = 0$
   Hence, the given equation is a quadratic equation.

10. Given equation is $5x^2 + 5x + 6 = 0$
    Here a = 5, b = 5, c = 6
    $D = b^2 - 4ac = (5)^2 - 4 \times 5 \times 6 = -95$

11. Let the smaller number be x and the larger number be y.
    Also, Square of the larger number(y^2) =18x
    According to question,
    $x^2 + y^2 = 208$
    $\Rightarrow x^2 + 18x = 208$
    $\Rightarrow x^2 + 18x - 208 = 0$
    $\Rightarrow x^2 + 26x - 8x - 208 = 0$
    $\Rightarrow (x + 26)(x - 8) = 0 \Rightarrow x = 8, x = -26$
    But, the numbers are positive. Therefore, x =8
    Square of the larger number = 18x = $18 \times 8 = 144$
    Therefore, larger number = $\sqrt{144}$ = 12
    Hence, the numbers are 8 and 12.

12. $3x^2 - 2\sqrt{6}x + 2 = 0$

$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$

$= \sqrt{3}x\left(\sqrt{3}x - \sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}x - \sqrt{2}\right)$

$= \left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right)$

$\left(\sqrt{3}x - \sqrt{2}\right)\left(\sqrt{3}x - \sqrt{2}\right) = 0$

$\Rightarrow \sqrt{3}x - \sqrt{2} = 0$ or $x = \sqrt{\frac{2}{3}}$.

$\therefore$ the roots are $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

13. Let the speed of the slow train be x km/hr

Then, the speed of the fast train = (x+10) km/hr

As we know that Time = $\frac{Distance}{Speed}$

Time taken by the fast train to cover 600 km = $\frac{600}{x+10}$ hrs

Time taken by the slow train to cover 600 km = $\frac{600}{x}$ hrs

$\therefore \frac{600}{x} - \frac{600}{x+10} = 3$

$\implies \frac{600(x+10)-600x}{x(x+10)} = 3$

$\implies \frac{6000}{x^2+10x} = 3$

$\implies 3x^2 + 30x - 6000 = 0$

$\implies 3(x^2 + 10x - 2000) = 0\, or\, x^2 + 10x - 2000 = 0$

$\implies x^2 + 50x - 40x - 2000 = 0$

$\implies x(x+50) - 40(x+50) = 0$

$\implies (x+50)(x-40) = 0$

Either x = -50 or x = 40

But the speed of the train cannot be negative. So, x= 40

Hence, the speed of the two trains are 40km/hr and 50km/hr respectively.

14. Let first number be x.

Then, second number = x + 4

According to the question,

$\frac{1}{x} - \frac{1}{x+4} = \frac{4}{21}$

$\frac{x + 4 - x}{x(x+4)} = \frac{4}{21}$

$\frac{4}{x^2 + 4x} = \frac{4}{21}$

$4x^2 + 4x = 84$

$\Rightarrow 4x^2 + 16x - 84 = 0$

$\Rightarrow 4(x^2 + 4x - 21) = 0$

$\Rightarrow (x^2 + 4x - 21) = 0$

$\Rightarrow (x + 7)(x - 3) = 0$

$\Rightarrow x + 7 = 0$ or $x - 3 = 0$

$\Rightarrow x = -7$ or $x = 3$

Therefore, the two numbers are 3 and 7 or -7 and -3

15. Let the number of hours required by the second pipe alone to fill the pool be x hrs.

    Then, the first and third pipe takes (x+5)hrs, (x-4) hrs respectively to fill the pool.

    So, the parts of the pool filled by the first, second and third pipes in one hour are respectively $\frac{1}{x+5}, \frac{1}{x}\, and \frac{1}{x-4}$

    Let the time taken by first and second pipes to fill the pool simultaneously be t hrs.

    Then the third pipe also takes the same time to fill the pool

    $\Longrightarrow \frac{1}{x+5} + \frac{1}{x} = \frac{1}{x-4}$

    $\Longrightarrow \frac{x+x+5}{x(x+5)} = \frac{1}{x-4}$

    $\Longrightarrow (2x+5)(x-4) = x^2 + 5x$

    $\Longrightarrow 2x^2 - 3x - 20 - x^2 - 5x = 0$

    $\Longrightarrow x^2 - 8x - 20 = 0$

    $\Longrightarrow x^2 - 10x + 2x - 20 = 0$

    $\Longrightarrow x(x-10) + 2(x-10) = 0$

    $\Longrightarrow (x-10)(x+2) = 0$

    Either $x - 10 = 0$ or $x + 2 = 0$

    $\Longrightarrow x = 10, -2$

    Since time taken cannot be negative. So $x = 10$

    Hence, the time required by the first, second and the third pipes to fill the pool individually are 15 hrs, 10 hrs and 6 hrs respectively.

16. Sol : Let the required number be x and x + 3.

    Then, according to given question we have,

    $x \times (x + 3) = 504$

    $\Rightarrow x^2 + 3x = 504$

$\Rightarrow x^2 + 3x - 504 = 0$

$\Rightarrow x^2 + 24x - 21x - 504 = 0$

$\Rightarrow x(x + 24) - 21(x + 24) = 0$

$\Rightarrow (x + 24)(x - 21) = 0$

$\Rightarrow x + 24 = 0$ or $x - 21 = 0$

$\Rightarrow x = - 24$ or $x = 21$

Case I: When x = -24

$\therefore x + 3 = -24 + 3 = -21$

Case II: When x = 21

$\therefore x + 3 = 21 + 3 = 24$

Hence, the numbers are -21, -24 or 21, 24.

17. n(n + 1) = 420 ...Given

$\Rightarrow n^2 + n = 420$

$\Rightarrow n^2 + n - 420 = 0$

Comparing with An2 + Bn + C = 0, we get

A = 1, B = 1, C = -420

Using the quadratic formula, $n = \frac{-B\pm\sqrt{B^2-4AC}}{2A}$

we get $\Rightarrow \frac{-1\pm\sqrt{1+1680}}{2} = \frac{-1\pm\sqrt{1681}}{2}$

$= \frac{-1\pm41}{2} = \frac{-1+41}{2}, \frac{-1-41}{2} = 20, -21$

n = -21 is in admissible as n is the number of terms.

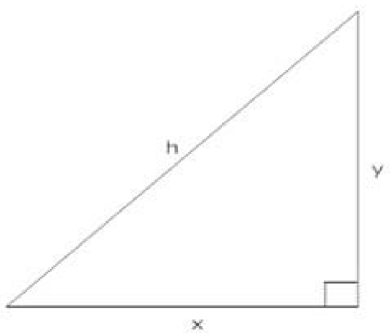
$\therefore n = 20$

Hence, the required value of n is 20.

18. Let base = x

Altitude = y

Hypotenuse = h



According to question,

h = x + 2

h = 2y + 1

$\Rightarrow$ x + 2 = 2y + 1

$\Rightarrow$ x + 2 - 1 = 2y

$\Rightarrow$ x - 1 = 2y

$\Rightarrow \frac{x-1}{2} = y$

And $x^2 + y^2 = h^2$

$\Rightarrow x^2 + \left(\frac{x-1}{2}\right)^2 = (x+2)^2$

$\Rightarrow$ x^2 - 15x + x - 15 = 0

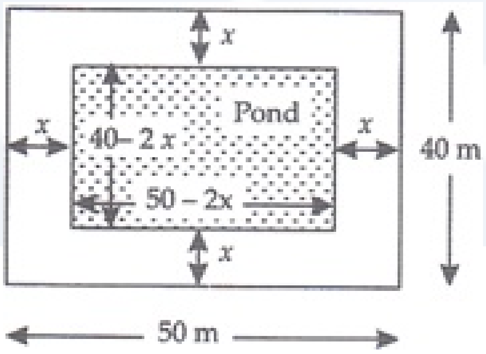
$\Rightarrow$ x^2 - 15x + x - 15 = 0

$\Rightarrow$ (x - 15)(x + 1) = 0

$\Rightarrow$ x = 15 or x = -1

Base = 15 cm

Altitude = $\frac{x+1}{2} = 8\text{cm}$

19. 

Let width of the pond be x m. Then,

The length of pond = $(50-2x)m$ and the breadth of pond = $(40-2x)m$

Area of grass around the pond = 1184 m^2

$\Rightarrow$ Area of Lawn - Area of Pond = 1184

$\Rightarrow$ 50 $\times$ 40 - (50 - 2x)(40 - 2x) = 1184

$\Rightarrow$ 2000 - (2000 - 100x - 80x + $4x^2$) - 1184 = 0

$\Rightarrow$ 2000 - (2000 - 180x + $4x^2$) - 1184 = 0

$\Rightarrow$ 2000 - 2000 + 180x - $4x^2$ - 1184 = 0

$\Rightarrow$ $4x^2$ - 180x + 1184 = 0

$\Rightarrow 4(x^2 - 45x + 296) = 0$

$\Rightarrow x^2 - 45x + 296 = 0$

Factorise now,

$\Rightarrow x^2 - 37x - 8x + 296 = 0$

$\Rightarrow x(x - 37) - 8(x - 37) = 0$

$\Rightarrow (x - 37)(x - 8) = 0$

$\Rightarrow x - 37 = 0$ or $x - 8 = 0$

$\Rightarrow x = 37$ or $x = 8$

When $x = 37$, then

The length of pond = $50 - 2 \times 37$

$= 50 - 74$

$= -24$ m (Length cannot be negative)

When $x = 8$, then

The length of pond = $50 – 2x$

$= 50 - 2 \times 8$

$= 50 - 16 = 34$ m

And the breadth of the pond

$= 40 - 2x$

$= 40 - 2 \times 8$

$= 40 - 16 = 24$ m

Therefore, the length and breadth of the pond are 34 m and 24 m respectively.

20. Let the original speed of the train be x km an hour

Then, the total speed taken by the train to travel a distance of 300 km at a uniform speed of x km an hour = $\frac{300}{x}$ hours

Increased speed of the train = $(x + 5)$ km an hour

Time taken by the train to travel a distance of 300 km at the increased speed = $\frac{300}{x+5}$ hours

According to the equation, $\frac{300}{x} - 2 = \frac{300}{x+5}$

$\Rightarrow \frac{300}{x} - \frac{300}{x+5} = 2 \Rightarrow 300\left(\frac{1}{x} - \frac{1}{x+5}\right) = 2$

$\Rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{2}{300} \Rightarrow \frac{1}{x} - \frac{1}{x+5} = \frac{1}{150}$

$\Rightarrow \frac{x+5-x}{x(x+5)} = \frac{1}{150} \Rightarrow \frac{5}{x^2+5x} = \frac{1}{150}$

$$\Rightarrow x^2 + 5x = 750$$
$$\Rightarrow x^2 + 5x - 750 = 0$$

Comparing with $ax^2 + bx + c = 0$

a = 1, b = 5, c = -750

Using the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

we get $= \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-750)}}{2(1)}$

$= \frac{-5 \pm \sqrt{25 + 3000}}{2} = \frac{-5 \pm \sqrt{3025}}{2} = \frac{-5 \pm 55}{2}$

$= \frac{-5+55}{2}, \frac{-5-55}{2} = 25, -30$

x = - 30 is inadmissible as x is the speed of the train and speed cannot be negative.

$\therefore$ x = 25

Hence, the original speed of the train is 25 km an hour.