CBSE Test Paper 02

Chapter 2 polynomials

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1. If 'α' and 'β' are the zeroes of the polynomial x^2 -6x + 8, then the value of $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$ is **(1)**
    a. 8
    b. 6
    c. 12
    d. 9
2. A polynomial of degree ____ is called a linear polynomial. **(1)**
    a. 1
    b. 3
    c. 2
    d. 0
3. If $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, then the other zeroes are **(1)**
    a. -2 and $-\frac{1}{2}$
    b. 2 and- $\frac{1}{2}$
    c. $\frac{1}{2}$ and $-\frac{1}{2}$
    d. 1 and $\frac{1}{2}$
4. If 'α' and 'β' are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then $\alpha+\beta=$ **(1)**
    1. $\frac{-b}{a}$
    2. $\frac{-c}{a}$
    3. $\frac{c}{a}$
    4. $\frac{b}{a}$
5. The degree of a biquadratic polynomial is **(1)**
    1. 2
    2. 4
    3. 3
    4. 1
6. Find a quadratic polynomial, whose sum and product of zeros are -5 and 6 respectively. **(1)**

7. Sum and product of zeroes of a quadratic polynomial are 0 and $\sqrt{15}$ respectively. Find the quadratic polynomial. **(1)**
8. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 - 5x + 4, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}-2\alpha\beta$. **(1)**
9. If one zero of $2x^2-3x+k$ is reciprocal to the other, then find the value of k **(1)**
10. If α and β are the zeroes of the quadratic polynomial p(x) = x^2 - p(x + 1) - c such that $(\alpha+1)(\beta+1)$= 0, what is the value of c? **(1)**
11. Find the value of b for which the polynomial $2x^3+9x^2-x-b$ is divisible by $2x+3$ **(2)**
12. α,β are zeroes of the quadratic polynomial x^2 - (k + 6)x + 2(2k - 1). Find the value of k if $\alpha+\beta=\frac{1}{2}\alpha\beta$. **(2)**
13. If α and β are the zeros of the polynomial f(x) = $5x^2$ - 7x + 1, find the value of $\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$. **(2)**
14. If α and β are the zeros of the quadratic polynomial f(x) = $3x^2$ - 4x + 1, find a quadratic polynomial whose zeros are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. **(3)**
15. Obtain all zeros of the polynomial ($2x^3$ - 4x - x^2 + 2), if two of its zeros are $\sqrt{2}$ and -$\sqrt{2}$ **(3)**
16. Find the zeroes of the quadratic polynomial $5x^2$ + 8x - 4 and verify the relationship between the zeroes and the coefficients of the polynomial. **(3)**
17. Find the zeroes of the polynomial y^2 + $\frac{3}{2}\sqrt{5}y$ - 5 by factorisation method and verify the relationship between the zeroes and coefficient of the polynomials. **(3)**
18. If two zeroes of the polynomial p(x) = x^4 - $6x^3$ - $26x^2$ + 138x - 35 are 2 ± $\sqrt{3}$. Find the other zeroes. **(4)**
19. Given that x - $\sqrt{5}$ is a factor of the polynomial x^3 - $3\sqrt{5}x^2$ - 5x + $15\sqrt{5}$, find all the zeroes of the polynomial. **(4)**
20. If the polynomial x^4 - $6x^3$ + $16x^2$ - 25x + 10 is divided by (x^2 - 2x + k), the remainder comes out to be x + a, find k and a. **(4)**

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Solution

1. d. 9

   Explanation: Here $a = 1, b = -6, c = 8, \alpha + \beta = 6, \alpha\beta = 8$

   Since $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)\left[\alpha^2+\beta^2-\alpha\beta\right]}{\alpha\beta} = \frac{(\alpha+\beta)\left[\alpha^2+\beta^2+2\alpha\beta-3\alpha\beta\right]}{\alpha\beta}$

   = $\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]}{\alpha\beta}$

   = $\frac{6[6^2-3\times8]}{8}$ = 9

2. a. 1

   Explanation: A polynomial of degree 1 is called a linear polynomial. Example 4x + 3, 65y are linear polynomials.

3. d. 1 and $\frac{1}{2}$

   Explanation: Since $\sqrt{2}$ and $-\sqrt{2}$ are the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, then $\left(x - \sqrt{2}\right)$ and $\left(x + \sqrt{2}\right)$ are the factors of given polynomial i.e., $\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) = \left(x^2 - 2\right)$ is a factor of given polynomial.

   $\therefore p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2 \Rightarrow p(x) = \left(x^2 - 2\right)\left(2x^2 - 3x + 1\right)$

   $$\begin{array}{r|l} & 2x^2 - 3x + 1 \\ x^2 - 2 & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\ & 2x^4 \qquad\quad - 4x^2 \\ & - \qquad\qquad + \\ \hline & -3x^3 + x^2 + 6x - 2 \\ & -3x^3 \qquad + 6x \\ & + \qquad\quad - \\ \hline & x^2 - 2 \\ & x^2 - 2 \\ & - \quad + \\ \hline & 0 \end{array}$$

   $\Rightarrow p(x) = \left(x^2 - 2\right)\left[2x^2 - 2x - x + 1\right] \Rightarrow$
   $p(x) = \left(x^2 - 2\right)\left[2x(x - 1) - 1(x - 1)\right] \Rightarrow$
   $p(x) = \left(x^2 - 2\right)(x - 1)(2x - 1)$

   $\therefore$ Other zeroes are x - 1 = 0 and 2x - 1 = 0 $\Rightarrow$ x = 1 and $x = \frac{1}{2}$

4. a. $\frac{-b}{a}$

Explanation: If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$,

$\because$ Sum of the zeroes of a quadratic polynomial $ax^2 + bx + c$= $\frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

then $\alpha + \beta = \frac{-b}{a}$

5. b. 4

Explanation: Biquadratic polynomial is a polynomial of the fourth degree.

Biquadratic polynomial = $a\left(x^2\right)^2 + b(x)^2 + c$ = $ax^4 + bx^2 + c$

6. Let α and β be the zeros of the required polynomial.

Then, (α + β) = -5 and $\alpha\beta$ = 6

f(x) = x^2- (α + β)x + $\alpha\beta$ = x^2 - (-5)x + 6

= x^2 + 5x + 6.

Hence, the required polynomial is f(x) = x^2 + 5x + 6.

7. Here sum of zeroes, $S = 0$

Product of zeroes, $P = \sqrt{15}$

Quadratic polynomial $p(x) = x^2 - (S)x + P$

= $x^2 - 0x +$ $\sqrt{15}$

= $x^2 +$ $\sqrt{15}$

8. We have, α and β are the roots of the quadratic polynomial. f(x) =x^2 - 5x + 4

Sum of zeros: $\alpha + \beta = -\frac{b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$

product of zeros: $\alpha\beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

We have a=1,b=-5 and c= 4.

Sum of the roots = α + β = 5

Product of the roots = αβ = 4

So,

$\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

$5/4 - 2 \times 4 = 5/4 - 8$ =$(5 - 32)/4$=$-27/4$

Hence,we get the result of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ = $-\frac{27}{4}$

9. Let be the two zeroes of the given polynomial.

Then, $\alpha \times \frac{1}{\alpha} = \frac{Constant\_term}{Coefficient(x^2)}$

$\Rightarrow 1 = \frac{k}{2}$
$\Rightarrow k = 2$

10. It is given that:

    p(x) = x^2 - px - p - c

    Here a = 1, b = - p and c = -p - c

    $\therefore \quad \alpha + \beta = p$ and $\alpha\beta = (-p - c)$

    $\because \quad (\alpha + 1)(\beta + 1) = 0$

    $\Rightarrow \quad \alpha\beta + \alpha + \beta + 1 = 0$

    $\Rightarrow$ - p - c + p + 1 = 0

    $\Rightarrow$ c = 1

11.

```
                 x² + 3x − 5
2x + 3 ) 2x³ + 9x² − x − b
         2x³ + 3x²
         −     −
         ----------
              6x² − x − b
              6x² + 9x
              −     −
         ----------
                  −10x − b
                  −10x − 15
                  +      +
         -----------
                  15 − b
```

    If the polynomial $2x^3 + 9x^2 - x - b$ is divisible by 2x + 3, then the remainder must be zero.

    So, 15 - b = 0, b = 15

12. Polynomial is $x^2 - (k + 6)x + 2(2k - 1)$.

    $\alpha + \beta = -\frac{\mathrm{b}}{\mathrm{a}} = \frac{\mathrm{k}+6}{1} = \mathrm{k} + 6$

    and $\alpha\beta = \frac{\mathrm{c}}{\mathrm{a}} = \frac{2(2\mathrm{k}-1)}{1} = 4\mathrm{k} - 2$

    Now, $\alpha + \beta = \frac{1}{2}\alpha\beta$

    k + 6 = $\frac{1}{2}$(4k - 2)

    k+6 = 2k -1

    k = 7

13. Compare f(x) = $5x^2 - 7x + 1$ with $ax^2 + bx + c$ we get,

a = 5, b = - 7 and c= 1

Since α and β are the zeros of $5x^2 - 7x + 1$, we have

$$\alpha+\beta=-\frac{(b)}{a}=-\frac{(-7)}{5}=\frac{7}{5}$$
$$\alpha\beta=\frac{c}{a}=\frac{1}{5}$$
$$\therefore \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$$
$$=\frac{\frac{7}{5}}{\frac{1}{5}}$$
$$=\frac{7}{5}\times\frac{5}{1}$$
= 7

14. Here it is given that the zeros of f(x)= $3x^2$-4x+1 are $\alpha and \beta$

Here a=3, b=-4 and c=1

$$\alpha+\beta=-\frac{\mathrm{b}}{\mathrm{a}}=-\left(-\frac{4}{3}\right)=\frac{4}{3} \text{ and } \alpha\beta=\frac{\mathrm{c}}{\mathrm{a}}=\frac{1}{3}$$

Let S and P denote respectively the sum and product of the zeros of the polynomial whose zeroes are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$, then

S = $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}=\frac{\alpha^3+\beta^3}{\alpha\beta}$ = $\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}=\frac{\left(\frac{4}{3}\right)^3-3\times\frac{1}{3}\times\frac{4}{3}}{\frac{1}{3}}$ = $\frac{28}{9}$ ....(1)

and, P = $\frac{\alpha^2}{\beta}\times\frac{\beta^2}{\alpha}=\alpha\beta=\frac{1}{3}$................(2)

Hence the polynomial with zeros $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ is

g(x)=x^2-Px+S=0

putting values of P and S from (1) and (2) we get the polynomial

$$\mathrm{g(x)}=\mathrm{x}^2-\frac{28}{9}\mathrm{x}+\frac{1}{3}$$
or $\mathrm{g(x)}=9\mathrm{x}^2$-28x+3

15. The given polynomial is:

f(x) = $2x^3 - x^2 - 4x + 2$.

It is given that the two zeroes of the above polynomial are $\sqrt{2}$ and -$\sqrt{2}$

Therefore, (x - $\sqrt{2}$)(x + $\sqrt{2}$) = (x^2 - 2) is a factor of f(x).

Now we divide (x) = $2x^3 - x^2 - 4x + 2$ by (x^2- 2), we obtain

$$
\begin{array}{l}
x^2-2\overline{)2x^3-x^2-4x+2}(2x-1 \\
\quad\quad\underline{2x^3 \quad\quad -4x} \\
\quad\quad\quad -x^2 \quad\quad +2 \\
\quad\quad\quad \underline{-x^2 \quad\quad +2} \\
\quad\quad\quad\quad\quad \times
\end{array}
$$

Where quotient = (2x - 1)

$\therefore$ f(x) = 0 $\Rightarrow$ (x^2 - 2)(2x - 1) = 0

$\Rightarrow$ (x - $\sqrt{2}$) (x + $\sqrt{2}$)(2x - 1) = 0

$\Rightarrow$ (x - $\sqrt{2}$) = 0 or (x + $\sqrt{2}$) = 0 or (2 x -1) = 0

$\Rightarrow$ x = $\sqrt{2}$ or x = - $\sqrt{2}$ or x = $\frac{1}{2}$.

Hence, all zeros of f(x) are $\sqrt{2}$, -$\sqrt{2}$ and $\frac{1}{2}$.

16. p(x) = $5x^2$ + 8x - 4 = 0

= $5x^2$ + 10x - 2x - 4 = 0

= 5x(x + 2) - 2(x + 2) = 0

= (x + 2)(5x - 2) = 0

Hence, zeroes are -2 and $\frac{2}{5}$

Verification: Sum of zeroes = $-2+\frac{2}{5}=\frac{-8}{5}$

Product of zeroes = $(-2)\times\left(\frac{2}{5}\right)=\frac{-4}{5}$

Again sum of zeroes = $-\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2}=\frac{-8}{5}$

Product of zeroes = $\frac{\text{Constant term}}{\text{Coeff. of } x^2}$ = $\frac{-4}{5}$

Verified.

17. y^2 + $\frac{3}{2}\sqrt{5}y$ - 5= $\frac{1}{2}\left(2y^2+3\sqrt{5}y-10\right)$

$=\frac{1}{2}\left(2y^2+4\sqrt{5}y-\sqrt{5}y-10\right)$

$=\frac{1}{2}[2y(y+2\sqrt{5})-\sqrt{5}(y+2\sqrt{5})]$

$=\frac{1}{2}(y+2\sqrt{5})(2y-\sqrt{5})$

$\Rightarrow \quad y=-2\sqrt{5},\frac{\sqrt{5}}{2}$ are zeroes of the polynomial.

If given polynomial is y^2 + $\frac{3}{2}\sqrt{5}y$ - 5 then a = 1, b=$\frac{3}{2}\sqrt{5}$ and c = -5

Sum of zeroes = $-2\sqrt{5}+\frac{\sqrt{5}}{2}=\frac{-3\sqrt{5}}{2}$ .... (i)

Also, $\frac{-b}{a}=\frac{-3\sqrt{5}}{2}$ ------ (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Product of zeroes = $-2\sqrt{5} \times \frac{\sqrt{5}}{2} = -5$ .... (iii)

Also, $\frac{c}{a} = \frac{-5}{1} = -5$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

18. As 2 ± $\sqrt{3}$ are the zeroes of p(x), so x - (2 ± $\sqrt{3}$) are the factors of p(x) and the product of factors,

$\{x-(2+\sqrt{3})\}\{x-(2-\sqrt{3})\}$

$= \{(x-2)-\sqrt{3}\}\{(x-2)+\sqrt{3}\}$

$= (x-2)^2 - (\sqrt{3})^2$

= x^2 - 4x + 1

Dividing p(x) by x^2 - 4x + 1

$$
\begin{array}{r}
x^2 - 2x - 35 \\
x^2 - 4x + 1 \,\big)\, x^4 - 6x^3 - 26x^2 + 138x - 35 \\
x^4 - 4x^3 + x^2 \\
- \quad + \quad - \\
\hline
-2x^3 - 27x^2 + 138x \\
-2x^3 + 8x^2 - 2x \\
+ \quad - \quad + \\
\hline
-35x^2 + 140x - 35 \\
-35x^2 + 140x - 35 \\
+ \quad - \quad + \\
\hline
0 \\
\hline
\end{array}
$$

Factorising (x^2 - 2x - 35) we get

= (x + 5)(x - 7)

x = -5, 7

Hence, other two zeroes of p(x) are - 5 and 7.

19.

$$
\begin{array}{r}
x^2 - 2\sqrt{5}\,x - 15 \\
x - \sqrt{5} \,\big)\, x^3 - 3\sqrt{5}\,x^2 - 5x + 15\sqrt{5} \\
x^3 - \sqrt{5}\,x^2 \\
- \quad + \\
\hline
-2\sqrt{5}\,x^2 - 5x \\
-2\sqrt{5}\,x^2 + 10x \\
+ \quad - \\
\hline
-15x + 15\sqrt{5} \\
-15x + 15\sqrt{5} \\
+ \quad - \\
\hline
0 \\
\hline
\end{array}
$$

On factorising the quotient, we get

$$x^2 - 2\sqrt{5}x - 15 = x^2 - 3\sqrt{5}x + \sqrt{5}x - 15$$
$$= x(x - 3\sqrt{5}) + \sqrt{5}(x - 3\sqrt{5})$$
$$= (x + \sqrt{5})(x - 3\sqrt{5})$$
$$\therefore\ (x + \sqrt{5})(x - 3\sqrt{5}) = 0$$
$$\Rightarrow\ x = -\sqrt{5}, 3\sqrt{5}$$

Therefore, all the zeroes are $\sqrt{5}, -\sqrt{5}$ and $3\sqrt{5}$.

20.

$$\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 25x + 10 \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (25-4k)x + 10 \\
 & (8-k)x^2 - (16-2k)x + (8k-k^2) \\
 & - \quad + \quad - \\
\hline
 & (2k-9)x + (10-8k+k^2)
\end{array}$$

Given, remainder = x + a

On comparing the multiples of x

(2k - 9)x = 1

or, 2k - 9 = 1 or k = $\frac{10}{2}$ = 5

On putting this value of k into other portion of remainder, we get

and a = 10 - 8k + k^2 = 10 - 40 + 25 = -5