CBSE Test Paper 02

Chapter 12 Area Related to Circle

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1. The wheel of the engine of a train is $14\frac{2}{7}$ m in circumference makes 7 revolutions in 10 seconds. The speed of the train is **(1)**
   a. 18 km/hr
   b. 48 km/hr
   c. 36 km/hr
   d. 24 km/hr
2. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is ______ **(1)**
   a. 22 : 7
   b. 14 : 11
   c. 11 : 14
   d. 7 : 22
3. The circumference of a circle whose diameter is 4.2cm is **(1)**
   a. 13.2cm
   b. 4.2 cm
   c. 22 cm
   d. 11 cm
4. The area of a ring having 'R' as outer radius and 'r' as inner radius is **(1)**
   a. $\pi(R+r)$
   b. $\pi(R^2-r^2)$
   c. $\pi(R^2+r^2)$
   d. $\pi(R-r)$
5. If the area of a circle is 'A', radius of the circle is 'r' and its circumference is 'C', then **(1)**
   a. rC = 2A
   b. $AC=\frac{r^2}{4}$
   c. $\frac{C}{A}=\frac{r}{2}$
   d. $\frac{A}{r}=C$

6. Find the area (in cm^2) of the circle that can be inscribed in a square of side 8 cm. **(1)**

7. Find the area of the sector of a circle having radius 6 cm and of angle 30°. [Take π = 3.14.] **(1)**

8. What is the perimeter of the sector with radius 10.5 cm and sector angle 60^o? **(1)**

9. A sector of a circle of radius 8 cm contains an angle of 135. Find the area of the sector. **(1)**

10. The circumference of two circles are in the ratio 4 : 9. Find the ratio of their area. **(1)**

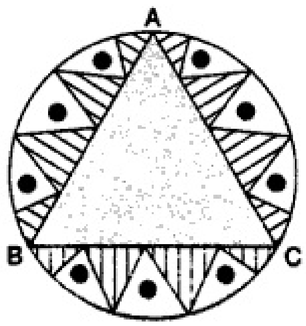
11. The wheels of a car make 2500 revolutions in covering a distance of 4.95 km. Find the diameter of a wheel. **(2)**

12. The area of a sector of a circle of radius 5 cm is $5\pi cm^2$. Find the angle contained by the sector. **(2)**

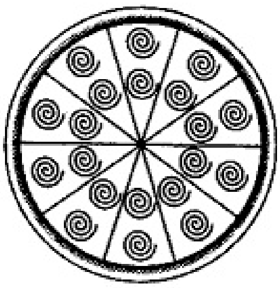
13. The area enclosed between the concentric circles is 770 cm^2. If the radius of the outer circle is 21 cm, find the radius of the inner circle. **(2)**

14. Prove that the area of a circular path of uniform width h surrounding a circular region of radius r is πh (2r + h). **(3)**

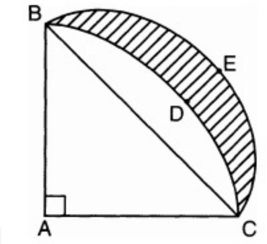
15. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in the figure. Find the area of the design (shaded region). **(3)**



16. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find:
    i. the total length of the silver wire required.
    ii. the area of each sector of the brooch. **(3)**



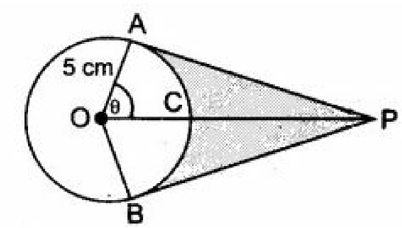
17. ABDC is a quadrant of a circle of radius 28 cm and a semi-circle BEC is drawn with BC as diameter. Find the area of the shaded region.$(\pi = \frac{22}{7})$ **(3)**



18. The inner perimeter of a racing track is 400 m and the outer perimeter is 488 m. The length of each straight portion is 90 m and the end are semicircles. Find the cost of developing the track at the rate of Rs 12.50/m^2. **(4)**

19. Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use π = 22/7). **(4)**

20. An elastic belt is placed around the rim of a pulley of radius 5 cm. From one point C on the belt, elastic belt is pulled directly away from the centre O of the pulley until it is at P, 10 cm from the point O. Find the length of the belt that is still in contact with the pulley. Also find the shaded area. **(4)**



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Solution

1. c. 36 km/hr

   Explanation: Given: Circumference of wheel = $14\frac{2}{7}=\frac{100}{7}$ m,

   No. of revolutions in 10 seconds = 7

   Now, No. of Revolutions = $\frac{\text{Total distance}}{\text{Circumference of wheel}}$

   $\Rightarrow 7=\frac{\text{Total distance}\times 7}{100}$

   $\Rightarrow$ Total distance in 10 second = 100 m

   $\therefore$ Distance in 1 hour = $\frac{100}{10}\times 3600$ = 36000 m = 36 km

   $\therefore$ Speed = 36 km/hr

2. b. 14 : 11

   Explanation: Let the radius of the circle be r and side of the square be $a.$ Then, according to question,

   $2\pi r=4a \Rightarrow a=\frac{2\pi r}{4}=\frac{\pi r}{2}$

   Now, ratio of their areas,

   $\frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2}$

   = $\frac{\pi r^2\times 4}{\pi^2 r^2}$

   = $\frac{14}{11}$

   $\Rightarrow \pi r^2 : a^2=14:11$

3. a. 13.2cm

   Explanation: Given: Diameter (d) = 4.2 cm

   $\therefore$ Circumference = πd = $\frac{22}{7}\times 4.2$ = 13.2 cm

4. b. $\pi(R^2-r^2)$

   Explanation: 

   The area of a ring having ‘R’ as outer radius and ‘r’ as inner radius is

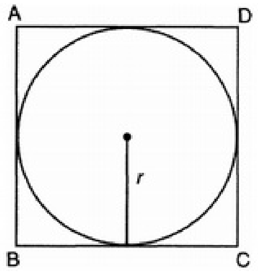
   $\pi \mathrm{R}^2-\pi r^2=\pi\left(\mathrm{R}^2-r^2\right)$

5. a. rC = 2A

Explanation: Here, Area of circle (A) = πr^2 and Circumference of circle (C)

$\Rightarrow$ rC = 2A

6. Side of square = 8 cm



Side of square = diameter of circle = 8 cm

$\therefore$ Radius of circle, r = $\frac{8}{2} = 4\text{cm}$

Area of circle = πr^2

= π $(4)^2$

$= \pi \times 4 \times 4$

$= 16\pi \text{ cm}^2$

So, Area of circle is $16\pi \text{ cm}^2$.

7. Radius of a circle = r = 6 cm

Central angle $= \theta = 30^\circ$

$\therefore$ Area of the sector $= \frac{\pi r^2 \theta}{360}$

$= \left(\frac{3.14 \times 6 \times 6 \times 30^\circ}{360^\circ}\right) \text{cm}^2$

= 9.42 cm^2

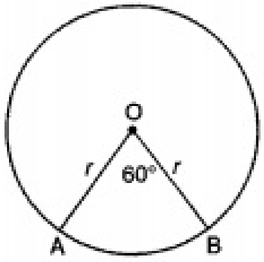
8. r = 10.5 cm , θ = 60^0

Perimeter of the sector = 2r + $\frac{2\pi r\theta}{360^\circ}$

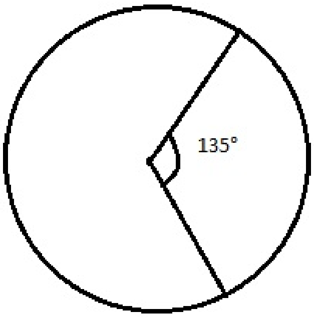
$= 10.5 \times 2 + 2 \times \frac{22}{7} \times \frac{10.5 \times 60}{360}$

$= 21 + 2 \times 22 \times 1.5 \times \frac{1}{6}$

$= 21 + 22 \times 1.5 \times \frac{1}{3}$



$= 21 + 11 = 32$ cm

9. 

It is given that the radius of circle = 8 cm

and angle, $\theta = 135^\circ$

Therefore, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{135^\circ}{360^\circ} \times \pi \times 8 \times 8$

$= \frac{135^\circ}{360^\circ} \times \pi \times 64$

$= 24\pi cm^2$

10. $\frac{2\pi r_1}{2\pi r_2} = \frac{4}{9} \Rightarrow \frac{r_1}{r_2} = \frac{4}{9}$

Now, $\frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Ratio of areas = 16 : 81

11. Distance covered by the wheel in 1 revolution

$= \left(\frac{4.95 \times 1000 \times 100}{2500}\right)$ cm

= 198 cm

$\therefore$ The circumference of the wheel = 198 cm

Let the diameter of the wheel be dcm

Then, $\pi d = 198$

$\Rightarrow \frac{22}{7} \times$ d = 198

$\Rightarrow \quad d = \frac{198 \times 7}{22}$

= 63 cm

Hence diameter of the wheel is 63 cm.

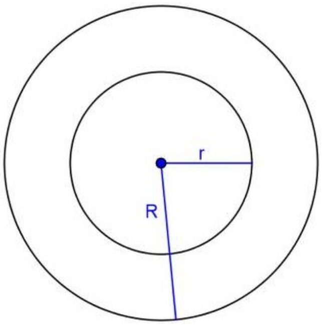
12. Radius of circle = 5cm

Area of sector $= 5\pi cm^2$

$\therefore$ Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$\Rightarrow 5\pi = \frac{\theta}{360^\circ} \times \pi \times 5 \times 5$

$\Rightarrow \theta = \frac{5\pi \times 360^\circ}{\pi \times 5 \times 5} = 72^\circ$

13. 

Radius of outer circle (R) = 21cm

let radius of inner circle = r cm

Given,

Area enclosed between concentric circle = 770cm^2

$\Rightarrow \pi R^2 - \pi r^2 = 770$

$\Rightarrow \frac{22}{7} \times 21 \times 21 - \frac{22}{7} \times r^2 = 770$

$\Rightarrow \frac{22}{7}\left(441 - r^2\right) = 770$

$\Rightarrow 441 - r^2 = \frac{770 \times 7}{22}$

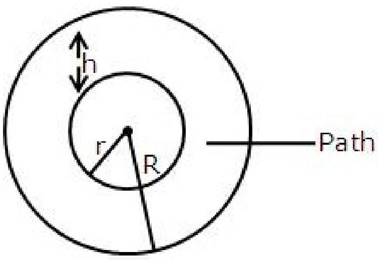
$\Rightarrow 441 - r^2 = 245$

$\Rightarrow r^2 = 441 - 245$

$\Rightarrow r^2 = 196$

$\Rightarrow r = \sqrt{196} = 14cm$

$\therefore$ radius of inner circle = 14cm

14. 

Given,

radius of inner circle = r

width of path = h

Then radius of outer circle = R

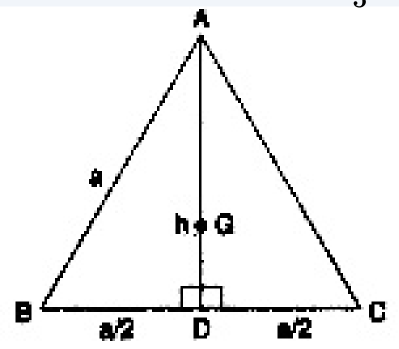
= r + h

$\therefore$ Area of path = Area of outer circle - Area of inner circle

$= \pi(R)^2 - \pi(r)^2$

$= \pi(r+h)^2 - \pi r^2$
$= \pi(r^2+h^2+2rh) - \pi r^2$
$= \pi r^2 + \pi h^2 + 2\pi rh - \pi r^2$
$= \pi h^2 + 2\pi rh$
$= \pi h(h+2r)$

15. Area of the design (shaded region)
= Area of the circular table cover - Area of the equilateral triangle ABC
$= \pi(32)^2 - \frac{\sqrt{3}}{4}a^2 - (1)$
Where a cm is the side of the equilateral triangle ABC.
Let h cm be the height of ΔABC
Since the centre of the circle coincides with the combined of the equilateral triangle.
Therefore, Radius of the
circumscribed circle = $\frac{2}{3}h\text{cm}$



According to the question,
$\frac{2}{3}h = 32 \Rightarrow h = 48$
Again, $a^2 = h^2 + \left(\frac{a}{2}\right)^2$ .... By Pythagoras theorem
$\Rightarrow a^2 = h^2 + \frac{a^2}{4}$
$\Rightarrow \mathrm{a}^2 - \frac{\mathrm{a}^2}{4} = \mathrm{h}^2 = \frac{3\mathrm{a}^2}{4} = \mathrm{h}^2$
$\Rightarrow a^2 = \frac{4h^2}{3} \Rightarrow a^2 = \frac{4(48)^2}{3}$
$\Rightarrow \mathrm{a}^2 = 3072 \Rightarrow \mathrm{a} = \sqrt{3072}$
$\therefore$ Form (1)
Required area $= \pi(32)^2 - \frac{\sqrt{3}}{4}(3072)$
$= \frac{22}{7}(1024) - 768\sqrt{3} = \left(\frac{22528}{7} - 768\sqrt{3}\right)\text{ cm}^2$

16. i. $\because$ Diameter = 35 mm
$\therefore$ Radius = $\frac{35}{2}$mm

$\therefore$ Circum ference = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{35}{2}$ = 110mm ..... (1)

Length of 5 diameters

$= 35 \times 5$ = 175 mm ...... (2)

$\therefore$ The total length of the silver wire required

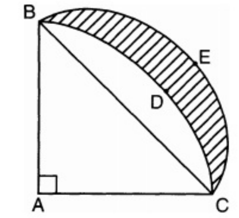
= 110 + 175 = 285 mm

ii. $r = \frac{35}{2} mm, \theta = \frac{360^\circ}{10} = 36^\circ$

$\therefore$ The area of each sector of the brooch

$= \frac{\theta}{360} \times \pi r^2$

$= \frac{36}{360} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = \frac{385}{4} \text{mm}^2$

17. 

As ABC is a quadrant of the circle, $\angle BAC$ will be measured 90^o.

In $\triangle ABC$, $BC^2 = AC^2 + AB^2$

$= (28)^2 + (28)^2$

$= 2(28)^2$

$\therefore \quad BC = 28\sqrt{2}\text{cm}$

Radius of semi-circle drawn on BC = $\frac{28\sqrt{2}}{2}$

$= 14\sqrt{2}$

Area of Semi-circle = $\frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times (14\sqrt{2})^2$

$= \frac{11}{7} \times 196(2)$

$= 11 \times 28 \times 2$

= 616 cm^2

Area of $\triangle ABC$ = $\frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 28 \times 28$

$= 14 \times 28$

$= = 392\text{cm}^2$

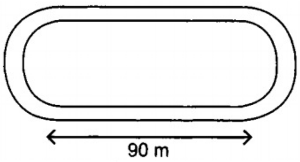
Area of of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

$= \frac{11}{7} \times 14 \times 28$
$= 11 \times 2 \times 28$

= 616 cm^2

Area of the shaded region = Area of semi-circle + Area of $\triangle$ - Area of quadrant

= 616 + 392 - 616 = 392 cm^2.

18. 

Perimeter of 2 inner semicircles = (400 - 2 $\times$ 90) m = (400 - 180) m = 220 m
Radius of each semicircle = $\frac{220}{2\pi} = \frac{220 \times 7}{2 \times 22}$ = 35 m
Perimeter of 2 outer semicircles = (488 - 180) m = 308 m
Radius of each outer semicircle = $\frac{308}{2\pi} = \frac{308 \times 7}{2 \times 22}$ = 49 m
Width of the track = outer radius - inner radius = 49 - 35 = 14 m
Area of rectangular tracks = 2 x area of rectangle
$= 2 \times l \times b = 2 \times 90 \times 14$
$= 28 \times 90$ = 2520 m^2
Area of two semicircular rings = area of one circular ring
$= \pi \left(\mathrm{R}^2 - r^2\right) = \frac{22}{7}\left(49^2 - 35^2\right) \mathrm{m}^2$
$= \frac{22}{7} \times (49 - 35)(49 + 35) \mathrm{m}^2$
$= \frac{22}{7} \times 14 \times 84 \mathrm{m}^2 = 44 \times 84$ = 3969 m^2
Total area of track = (2520 + 3696)m^2 = 6216 m^2
Cost of developing the track at the rate of Rs 12.50/ m^2 = Rs 6216 $\times$ 12.50 = Rs 77,700.

19. We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.
For the first triangle, we have a = 35, b = 53 and c = 66.
$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+53+66}{2} = 77 \mathrm{cm}$
Let Δ_1 be the area of the first triangle. Then,
$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)}$
$\Rightarrow \quad \Delta_1 = \sqrt{77(77-35)(77-53)(77-66)} = \sqrt{77 \times 42 \times 24 \times 11}$
$\Rightarrow \quad \Delta_1 = \sqrt{7 \times 11 \times 7 \times 6 \times 6 \times 4 \times 11}$=

$\sqrt{7^2\times 11^2\times 6^2\times 2^2}=7\times 11\times 6\times 2=924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore\quad s=\frac{a+b+c}{2}=\frac{33+56+65}{2}=77\text{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow\quad \Delta_2=\sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow\quad \Delta_2=\sqrt{77\times 44\times 21\times 12}$=

$\sqrt{7\times 11\times 4\times 11\times 3\times 7\times 3\times 4}=\sqrt{7^2\times 11^2\times 4^2\times 3^2}$

$\Rightarrow\quad \Delta_2=7\times 11\times 4\times 3=924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

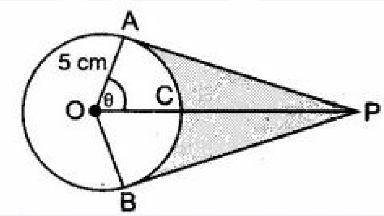
$\Rightarrow\quad \pi r^2=\Delta_1+\Delta_2$

$\Rightarrow\quad \pi r^2$= 924 + 924

$\Rightarrow\quad \frac{22}{7}\times r^2=1848$

$\Rightarrow\quad r^2=1848\times\frac{7}{22}=3\times 4\times 7\times 7\Rightarrow$

$r=\sqrt{3\times 2^2\times 7^2}=2\times 7\times\sqrt{3}=14\sqrt{3}\text{cm}$

20. 

$\cos\theta=\frac{1}{2}$ or, $\theta=60^\circ$

Reflex $\angle AOB=240^\circ$

$\therefore ADB=\frac{2\times 3.14\times 5\times 240}{360}=20.93\text{cm}$

Hence length of elastic in contact = 20.93 cm

Now, $AP=5\sqrt{3}\text{cm}$

a ($\triangle OAP$) = $\frac{1}{2}\times\text{base}\times\text{height}\ =\ \frac{1}{2}\times 5\times 5\sqrt{3}\ =\ \frac{25\sqrt{3}}{2}$

Area $(\Delta OAP+\Delta OBP)=2\times\ \frac{25\sqrt{3}}{2}=25\sqrt{3}=43.25\text{cm}^2$

Area of sector OACB = $\frac{\theta}{360}\times\pi\text{r}^2$

= $\frac{25\times 3.14\times 120}{360}=26.16\text{cm}^2$

Shaded Area = 43.25 - 26.16 = 17.09 cm^2