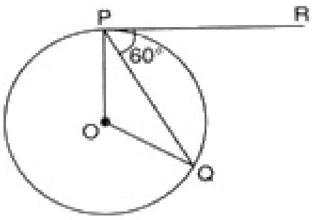
CBSE Test Paper 02

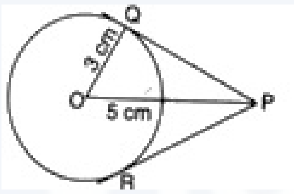
Chapter 10 Circles

1. If O is the centre of a circle, PQ is a chord and tangent PR at P makes an angle of 60° with PQ, then $\angle$POQ is equal to **(1)**



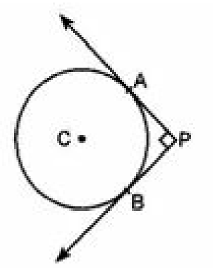
   a. 110°
   b. 120°
   c. 100°
   d. 90°

2. In the given figure, if OQ = 3 cm, OP = 5 m, then the length of PR is **(1)**



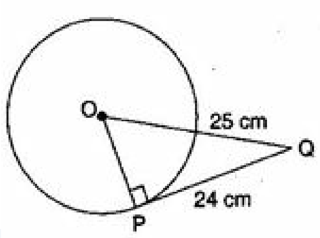
   a. 4 cm
   b. 3 cm
   c. 5 cm
   d. 6 cm

3. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA $\perp$ PB, then the length of each tangent is: **(1)**

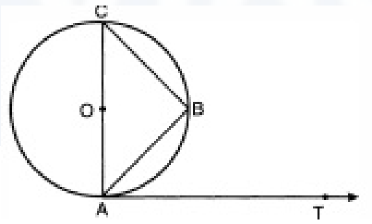


   a. 5 cm
   b. 3 cm
   c. 4 cm
   d. 8 cm

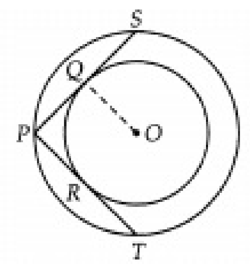
4. In the given figure, O is the centre of the circle with radius 10 cm. If $AB \parallel CD$, AB = 16 cm and CD = 12 cm, the distance between the two chords AB and CD is : **(1)**
   a. 12 cm
   b. 20 cm
   c. 16 cm
   d. 14 cm
5. The length of tangent PQ, from an external point Q is 24 cm. If the distance of the point Q from the centre is 25 cm, then the diameter of the circle is **(1)**



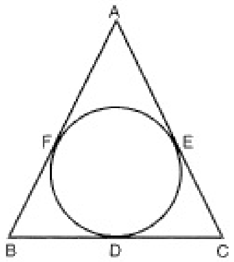
   a. 15 cm
   b. 14 cm
   c. 12 cm
   d. 7 cm
6. In the given figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^{0}$. If AT is the tangent to the circle at the point A, find $\angle BAT$ **(1)**



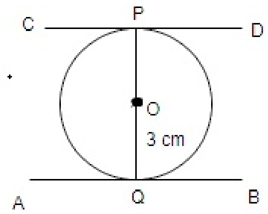
7. What term will you use for a line which intersect a circle at two distinct points? **(1)**
8. In the fig. there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If PR = 5 cm find the length of PS. **(1)**



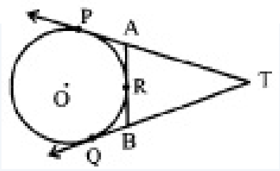
9. A triangle ABC is drawn to circumscribe a circle. If AB = 13 cm, BC = 14 cm and AE = 7 cm, then find AC. **(1)**



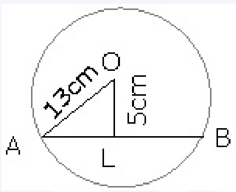
10. Find the distance between two parallel tangents of a circle of radius 3 cm. **(1)**



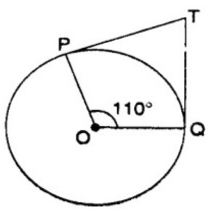
11. In the given figure, TP and TQ are tangents from T to the circle with centre O and R is any point on the circle. If AB is a tangent to the circle at R, prove that TA + AR = TB + BR. **(2)**



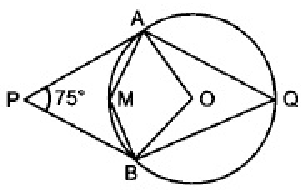
12. In figure, if OL = 5 cm, OA = 13 cm, then length of AB is **(2)**



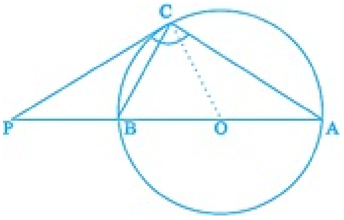
13. In figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^{\circ}$, then find $\angle PTQ$. **(2)**



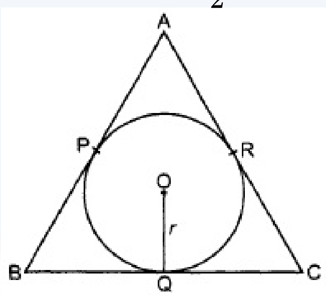
14. From an external point P, tangents PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA = 14 cm, find the perimeter of $\triangle PCD$. **(3)**
15. In the given figure, O is the centre of the circle. Determine $\angle AQB$ and $\angle AMB$, if PA and PB are tangents **(3)**



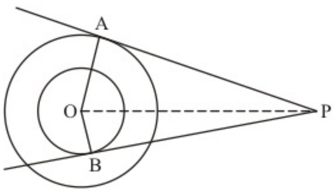
16. Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre. **(3)**
17. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110^0$, find $\angle CBA$.
[Hint: Join C with centre O]. **(3)**



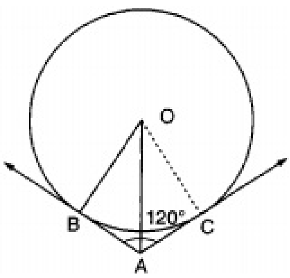
18. In figure, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius r at P, Q and R respectively. Prove that
    i. AB + CQ = AC + BQ
    ii. Area (ABC) = $\frac{1}{2}$ (perimeter of $\triangle$ABC) $\times$ r **(4)**



19. In Fig. there are two concentric circles with centre O of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP. **(4)**



20. In fig. two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^\circ$. Prove that OA = 2AB. **(4)**



CBSE Test Paper 02

Chapter 10 Circles

Soluiton

1. b. $120°$

   Explanation: Here $\angle RPO = 90°$

   $\angle RPQ = 60°$ (given)

   $\therefore \angle OPQ = 90° - 60° = 30°$ $\angle PQO = 30°$ Also [Opposite angles of equal radii] Now,
   In triangle OPQ,

   $$\angle OPQ + \angle PQO + \angle QOP = 180^\circ$$

   $\Rightarrow 30° + 30° + \angle QOP = 180°$

   $\Rightarrow \angle QOP = 120°$

2. a. 4 cm

   Explanation: Here $\angle Q = 90°$ [Angle between tangent and radius through the point of contact]

   Now, in right angled triangle OPQ,

   $OP^2 = OQ^2 + PQ^2$

   $\Rightarrow (5)^2 = (3)^2 + PQ^2$

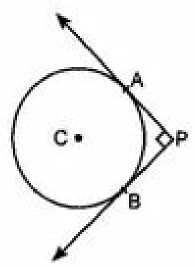
   $\Rightarrow PQ^2 = 25 - 9 = 16$

   $\Rightarrow PQ = 4$ cm

   But PQ = PR [Tangents from one point to a circle are equal]

   Therefore, PR = 4 cm

3. c. 4 cm

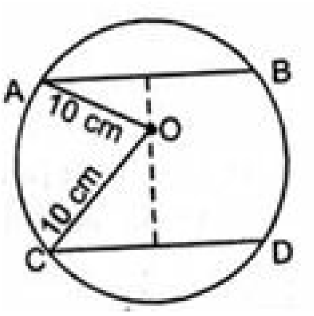
   Explanation: 

   Construction: Joined AC and BC. Here $CA \perp AP$ and $CB \perp BP$ and $PA \perp PB$ Also AP = PB

   Therefore, BPAC is a square. $\Rightarrow AP = PB = BC = 4$ cm

4. d. 14 cm

Explanation:



Let OP be the perpendicular to chord AB and P bisects the chord AB and OQ be the perpendicular to chord CD and Q bisects the chord CD.

$\therefore$ AP = BP = 8 cm and CQ = DQ = 6 cm

In triangle AOP, $OA^2 = OP^2 + AP^2$

$\Rightarrow (10)^2 = OP^2 + (8)^2$

$\Rightarrow OP^2 = 100 - 64 = 36$

$\Rightarrow$ OP = 6 cm

And in right angled triangle COQ,

$OC^2 = OQ^2 + CQ^2$

$\Rightarrow (10)^2 = OQ^2 + (6)^2$

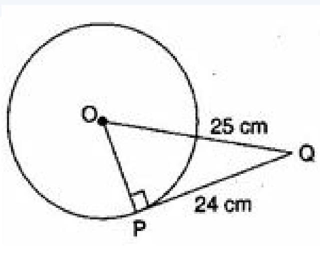
$\Rightarrow OQ^2 = 100 - 36 = 64$

$\Rightarrow$OQ = 8 cm

Therefore, distance between two chord AB and CD = OP + OQ = 6 + 8 = 14 cm

5. b. 14 cm

Explanation:



Here $\angle$OPQ = 90^o [Angle between tangent and radius through the point of contact]

$\therefore$ $OQ^2 = OP^2 + PQ^2 \Rightarrow (25)^2 = OP^2 + (24)^2$

$\Rightarrow OP^2 = 625 - 576 \Rightarrow OP^2 = 49$

$\Rightarrow OP^2 = 49 \Rightarrow$ OP = 7 cm

Therefore, the diameter = 2 $\times$ OP = 2 $\times$ 7 = 14 cm

6. $\because \angle ACB = 50^\circ$
   $\angle CBA = 90^\circ$ (Angle in semi-circle)
   $\therefore \angle OAB = 90^\circ - 50^\circ$
   $= 40^\circ$
   $\angle BAT = 90^\circ - \angle OAB$
   $= 90^\circ - 40^\circ$
   $= 50^\circ$

7. A line that interests a circle at two points in a circle is called a Secant.

8. PQ = PR = 5 cm (Length of Tangents from same external point are always equal)
   and PQ = QS (perpendicular from center of the circle to the chord bisects the chord)
   $\therefore$ PS = 2PQ
   $= 2 \times 5 = 10\text{cm}$

9. $AF = AE = 7cm$ (tangents from same external point are equal)
   $\therefore BF = AB - AF = 13 - 7 = 6cm$
   $BD = BF = 6cm$ (tangents from same external point)
   $\therefore CD = BC - BD = 14 - 6 = 8cm$
   $CE = CD = 8cm$
   $\therefore AC = AE + EC$
   $= 7 + 8 = 15cm.$

10. Distance between two parallel tangents = diameter = PQ
    PQ = OP + OQ = 3 + 3 = 6cm
    The total distance between two parallel tangents lines is 6 cm.

11. Length of tangents from same external point are equal.
    $\therefore$ TP = TQ
    AP = AR
    and BR = BQ
    We have, $TP = TQ$
    $\Rightarrow TA + AP = TB + BQ$
    $\Rightarrow TA + AR = TB + BR$
    Hence proved.

12. AB = 2 AL = $2\sqrt{OA^2 - OL^2}$

= $2\sqrt{13^2 - 5^2}$

= $2\sqrt{169 - 25}$ = $2\sqrt{144}$

= 2×12 = 24 cm

13. $\angle$POQ = 110^o

$\angle$OPT = 90^o [Angle between tangent and radius through the point of contact]

$\angle$OQT = 90^o [Angle between tangent and radius through the point of contact]

In quadrilateral OPTQ,

$\angle$POQ + $\angle$OQT + $\angle$PTQ = 360^o

$\because$ The sum of all the angles of a quadrilateral is 360^o.

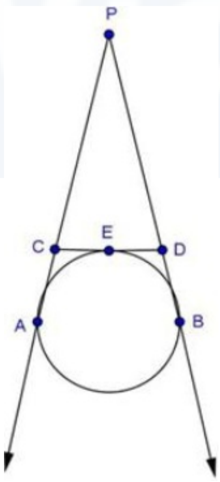
$\Rightarrow$ $110^o + 90^o + 90^o + \angle PTQ = 360^o$

$\Rightarrow$ $290^o + \angle PTQ = 360^o$

$\Rightarrow \angle PTQ = 360^o - 290^o = 70^o$

$\Rightarrow$ Hence, the $\angle$PTQ is 70^o.

14. We have,



AC = CE, BD = DE

And, AP = BP = 14 cm

$\therefore$ Perimeter of ΔPCD = PC + CD+ PD

$\Rightarrow$ Perimeter of ΔPCD = PC + (CE + ED) + PD

= (PC + CE) + (ED + PD)

= (PC + AC) + (BD + PD)

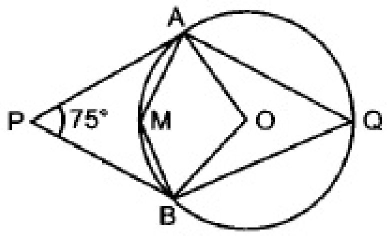
= PA + PB

= 14 + 14

= 28

$\therefore$ Perimeter of $\triangle$ PCD = 28 cm.

15. Given,



$OA \perp PA$ and

$OB \perp PB$

$\therefore \angle \mathrm{OAP} = 90^\circ$ and $\angle \mathrm{OBP} = 90^\circ$

$\Rightarrow \angle \mathrm{OAP} + \angle \mathrm{OBP} = 180^\circ$

PBOA is cyclic quadrilateral

$\therefore \angle APB + \angle AOB = 180^\circ$

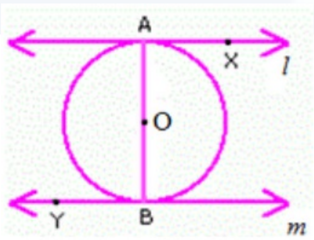
$\Rightarrow \angle AOB = 180^\circ - 75^\circ = 105^\circ$

Now $\angle AOB = 2\angle AQB$

$\therefore \angle \mathrm{AQB} = \frac{1}{2}(105^\circ) = 52.5^\circ$

Now $\angle AMB + \angle AQB = 180^\circ$

$\Rightarrow \angle AMB = 180^\circ - 52.5^\circ = 127.5^\circ$

16. 

Given: l and m are the tangent to a circle such that l || m, intersecting at A and B respectively.

To prove: AB is a diameter of the circle.

Proof:

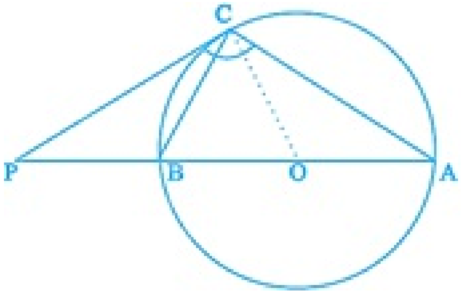
A tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \angle XAO = 90^\circ$

and $\angle YBO = 90^\circ$

Since $\angle XAO + \angle YBO = 180^{\circ}$
An angle on the same side of the transversal is 180°.
Hence the line AB passes through the centre and is the diameter of the circle.

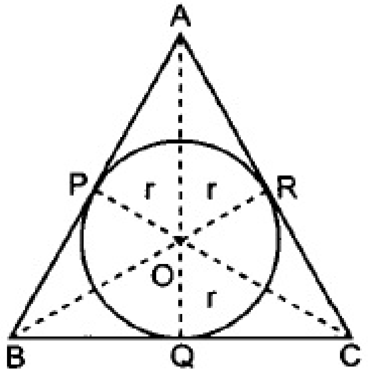
17. According to the question,we are given that tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle$PCA = 110^{0}, find $\angle$CBA.



OC and CP are radius and tangent respectively at contact point C.
So, $\angle$OCP = 90^{0}
$\angle$OCA = $\angle$ACP - $\angle$OCP
$\Rightarrow$ $\angle$OCA = 110^{0} - 90^{0}
$\Rightarrow$ $\angle$OCA = 20^{0}
In $\triangle$OAC,
OA = OC [Radii of same circle]
Therefore, $\angle$OCA = $\angle$A = 20^{0} [Since, Angles opposite to equal sides are equal]
CP and CB are tangent and chord of a circle.
Therefore, $\angle$CBP = $\angle$A [Angles in alternate segments are equal]
In ΔCAP,
$\angle$P + $\angle$ A + $\angle$ACP = 180^{0} [Angled sum property of a triangle]
$\Rightarrow$ $\angle$P + 20^{0} + 110^{0} = 180^{0}
$\Rightarrow$ $\angle$P = 180^{0} - 130^{0}
$\Rightarrow$ $\angle$P = 50^{0}
In $\triangle$BPC,
Exterior angle $\angle$CBA = $\angle$P + $\angle$BCP
$\Rightarrow$ $\angle$CBP = 50^{0} + 20^{0}
$\Rightarrow$ $\angle$CBP = 70^{0}

18. Given, the sides AB, BC and CA of triangle ABC touch a circle with centre O and radius

r at P, Q and R respectively.



(i) AP = AR [Tangents from A] ...(i)

Similarly, BP = BQ ...(ii)

CR = CQ ...(iii)

Now,

$\because$AP = AR

$\Rightarrow$ (AB-BP) = (AC - CR)

$\Rightarrow$ AB + CR = AC + BP

$\Rightarrow AB + CQ = AC + BQ$ [Using eq. (ii) and (iii)]

(ii) Let AB = x, BC = y, AC = z

$\therefore$Perimeter of $\triangle$ABC =x + y + z ...(iv)

Area of $\triangle$ABC = $\frac{1}{2}$ [area of $\triangle$AOB + area of $\triangle$BOC + area of $\triangle$AOC]

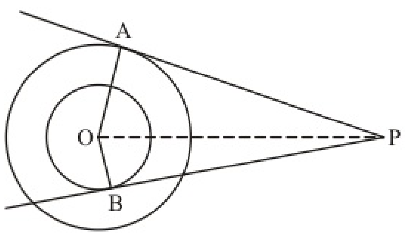
$\Rightarrow$ Area of $\triangle$ABC = $\frac{1}{2}\times$AB $\times$ OP + $\frac{1}{2}$ $\times$ BC $\times$ OQ + $\frac{1}{2}$ $\times$ AC $\times$ OR

$\Rightarrow$ Area of $\triangle$ABC = $\frac{1}{2}\times x \times r$+ $\frac{1}{2}$ $\times y \times r$+ $\frac{1}{2}\times z \times r$

$\Rightarrow$ Area of $\triangle$ABC = $\frac{1}{2}(x + y + z)$ $\times$ r

$\Rightarrow$ Area of $\triangle$ABC = $\frac{1}{2}$(Perimeter of $\triangle$ABC) $\times$ r

19.



Join OA, OB and OP.

$\angle OAP = 90^\circ$ as the tangent makes a right angle with the radius of the circle at the point of contact.

In $\triangle$OAP, we have,

$OP^2 = OA^2 + AP^2$

$\Rightarrow OP^2 = 5^2 + 12^2$

$\Rightarrow$ OP = 13 cm

$\angle OBP = 90^\circ$ as the tangent makes a right angle with the radius of the circle at the point of contact.

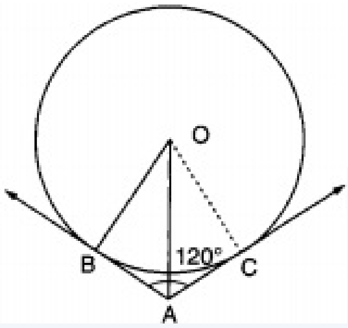
In $\triangle$OBP, we have,

$OP^2 = OB^2 + BP^2$

$\Rightarrow 13^2 = 3^2 + Bp^2$

$\Rightarrow BP^2 = 169 - 9 = 160$

$\Rightarrow$ BP = $\sqrt{160}$ cm = $4\sqrt{10}$ cm

20. 

In Δ's OAB and OAC, we have,

$\angle OBA = \angle OCA = 90^\circ$

OA = OA [Common]

AB = AC [$\because$ Tangents from an external point are equal in length]

Therefore, by RHS congruence criterion, we have,

$\Delta OBA \cong \Delta OCA$

$\Rightarrow \angle OAB = \angle OAC$ [By c.p.c.t.]

$\therefore \angle OAB = \angle OAC = \frac{1}{2}\angle BAC$

$= \frac{1}{2} \times 120^\circ = 60^\circ$

$\Rightarrow \angle OAB = \angle OAC = 60^\circ$

In ΔOBA, we have,

$\cos \mathrm{B} = \frac{AB}{OA}$

$\Rightarrow \cos 60^\circ = \frac{AB}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AB}{OA}$

$\Rightarrow$ OA = 2AB

Hence proved.